

# Quantum Distortion Gravity: A Fixed Point at $n_f = 3$ in the Einstein–Hilbert Flow with QDG-Like Matter

L. E. Pavese<sup>\*</sup> <sup>1</sup>

<sup>1</sup>Independent Researcher, Italy

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## Abstract

The first version of this note claimed, from a phenomenological flow, that Quantum Distortion Gravity (QDG) has no non-trivial fixed point for  $n_f = 3$  Dirac fermions. We withdraw that claim. The flow used there was never derived from the Einstein–Hilbert action, and its  $n_f = 3$  result depends on a matter coefficient that is inconsistent with the other coefficients of the same model. Here we derive the Einstein–Hilbert flow from the Wetterich equation (Litim cutoff, harmonic gauge, linear split,  $S^4$ ), add minimally coupled matter with the content  $n_f$  Dirac fermions plus two Proca fields in the massless limit, and reduce the fixed-point condition to a cubic equation. In this truncation (optimised cutoff, minimal-coupling counting of the matter fields) a non-Gaussian fixed point exists for every  $n_f < 10$ . For  $n_f = 3$  it sits at  $(\tilde{G}_*, \lambda_*) = (2.73, -0.318)$  with two real positive critical exponents (2.21, 0.72), and it exists for all nine regulators tested. Its position and the character of its exponents depend strongly on the regulator and on the matter-cutoff convention: for the optimised cutoff it lies outside the range  $\tilde{G}_* < 2$ ,  $\lambda_* > -1$  in which the truncation is customarily trusted, while the exponential regulators place it inside, with a complex pair of exponents. The Weyl-gauged operators and the distortion sector of QDG are not part of the flow. The result is therefore an existence statement in a truncation, not evidence that QDG is asymptotically safe.

**Keywords:** quantum gravity, metric-affine gravity, Weyl symmetry, asymptotic safety, functional renormalization group, fixed point, Proca fields

## 1 Introduction

Quantum Distortion Gravity (QDG) was introduced in Ref. [10] as a metric-affine theory with gauged Weyl symmetry. Ref. [11] showed that, in unitary gauge, its infrared limit is Einstein gravity coupled to two massive vector (Proca) fields, the Weyl vector  $V_\mu$  and the axial vector  $\tilde{a}_\mu$ , with eight propagating bosonic degrees of freedom. The renormalization-group (RG) behaviour of the theory at high energies was left open there. The first version of the present note [12] tried to settle one aspect of it, the existence of an ultraviolet fixed point at the physically interesting fermion number  $n_f = 3$ , and concluded that none exists.

That conclusion does not survive a re-examination, for two independent reasons that are developed in Sections 3–7.

1. The flow of Ref. [12] is a model. Its gravitational functions  $A(\lambda)$ ,  $B(\lambda)$  are mode-counting expressions of the *volume* term of the Wetterich trace that were used as if they were the coefficient of the curvature term, which is what determines the anomalous dimension. They

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<sup>\*</sup>Corresponding author e-mail: [lucapv02@gmail.com](mailto:lucapv02@gmail.com)

are not derived from the Einstein–Hilbert (EH) action, and in the pure-gravity limit the model does not reproduce the EH fixed point (Section 7).

2. Even inside that model, the absence of a fixed point at  $n_f = 3$  is produced by one matter coefficient,  $C_{\text{bl}} = 2(1 - n_f)/\pi$ , that is twice as large per fermion ( $-8$  instead of  $-4$  in the units of Eq. (8)) as the Dirac-fermion coefficient obtained from the heat-kernel counting. With the consistent value  $C_{\text{bl}} = (3 - 2n_f)/(2\pi)$  the model has a fixed point at  $n_f = 3$  (Section 7).

What can be said instead is obtained from the *derived* EH flow with minimally coupled matter, treated in the literature on the asymptotic-safety scenario [1] with matter [5, 6]. We derive that flow in Section 3 (the Hessian of the EH action is taken from Ref. [3]; everything else is computed here and reproduced by the scripts listed in Appendix A), reduce its fixed-point equations to a cubic (Proposition 3.1), and apply it to QDG-like content in Sections 4–6. The result is that a fixed point *does* exist for the QDG-like content at  $n_f = 3$ , but at a coupling that, for the optimised cutoff, lies outside the range in which the truncation is customarily trusted, and with a position that varies by a factor of five among the regulators tested.

**Logical status.** Following Refs. [10, 11], *Proposition* means proven within stated assumptions, *Assumption* denotes an input, and *Remark* or *Open Problem* means not proven. Section headings and tables carry the tags [DERIVED] (derived in this paper and reproduced by a script), [INPUT] (taken from a cited source that was checked), [MODEL] (phenomenological, not derived), and [OPEN] (not established).

## 2 Setting, content and assumptions

### 2.1 The matter content assigned to QDG

In unitary gauge  $\phi = \phi_0 = \sqrt{6} M_{\text{Pl}}$  of the dilaton, the infrared action of QDG is (signature  $(-, +, +, +)$ ;  $\hat{R}$  is the Levi-Civita curvature; Ref. [11] and the chapter on the unitary gauge in the second edition of Ref. [13])

$$S_{\text{IR}} = \int d^4x \sqrt{-g} \left[ \frac{\phi_0^2}{12} \hat{R} - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} - \frac{1}{2} m_V^2 \hat{V}_\mu \hat{V}^\mu - \frac{1}{4} \hat{f}_{\mu\nu} \hat{f}^{\mu\nu} - \frac{1}{2} m_a^2 \hat{a}_\mu \hat{a}^\mu \right. \\ \left. + \bar{\psi} (i \gamma^\mu \hat{\nabla}_\mu - y \phi_0) \psi + \frac{3}{2} g_a \hat{a}_\sigma \bar{\psi} \gamma^\sigma \gamma_5 \psi - \lambda_{\text{vac}} \phi_0^4 \right], \quad (1)$$

with  $m_V^2 = 2g_V^2 \phi_0^2$ ,  $m_a^2 = 12(\beta_1 + \beta_2) g_a^2 \phi_0^2$ . The algebraic distortion modes carry no degree of freedom at this order. The field content that matters for the RG flow of the Newton and cosmological constants is therefore: the metric, two massive vectors, and  $n_f$  Dirac fermions.

### 2.2 Assumptions of this note

**Assumption 2.1** (Truncation). The effective average action is  $\Gamma_k = \frac{1}{16\pi G_k} \int d^4x \sqrt{g} (-R + 2\bar{\Lambda}_k) + \Gamma_k^{\text{gf}} + \Gamma_k^{\text{gh}} + \Gamma_k^{\text{matter}}$  in Euclidean signature on a background  $S^4$ , with the linear split  $g = \bar{g} + h$ , harmonic gauge  $F_\mu = \bar{\nabla}^\nu h_{\mu\nu} - \frac{1}{2} \bar{\nabla}_\mu h$  with gauge parameter  $\alpha = 1$ , and the optimised (Litim) cutoff profile  $R^{(0)}(z) = (1 - z)\theta(1 - z)$ . Matter is minimally coupled with canonical kinetic terms; the matter wave-function renormalisations are not run.

**Assumption 2.2** (Proca fields in the massless limit). Each of the two massive vectors of Eq. (1) is counted as a massless vector plus one minimally coupled real scalar (its Stueckelberg mode). This is the limit  $m^2/k^2 \rightarrow 0$ . It is the relevant limit for a fixed point reached as  $k \rightarrow \infty$  only if the masses stay finite along the flow, which is itself an assumption (they are proportional to

$\phi_0$ , whose running is not computed). The limit ignores non-minimal couplings of the vectors and of the Stueckelberg scalar to curvature. The Yukawa coupling  $y$  and the axial coupling  $g_a$  of Eq. (1) are not run.

**Remark 2.3** (Why the Proca counting, and what it leaves out). The massless limit of the pair (dilaton, Weyl vector) of the Weyl-gauged theory has the field content of one Proca field: a massless vector (2 polarisations) plus a scalar (1), which is the same 3 that the unitary gauge gives to the massive  $\hat{V}_\mu$ . For  $\hat{a}_\mu$  the same count is an additional assumption. What is *not* contained in Assumption 2.2 is the dilaton's Weyl-covariant coupling to curvature, its derivative coupling to  $V_\mu$ , and every operator of the distortion sector. Whether these modify the flow is an open problem (Section 8.2).

### 2.3 Conventions

We write  $\tilde{G} = k^2 G_k$ ,  $\lambda = \bar{\Lambda}_k/k^2$  and  $t = \ln k$ . The anomalous dimension is  $\eta = \partial_t \ln G_k$ , so that  $\beta_{\tilde{G}} = \partial_t \tilde{G} = (2 + \eta)\tilde{G}$ . We use  $u \equiv 1 - 2\lambda$ . A real scalar, a Dirac fermion, a massless vector and a Proca field in the massless limit are counted by  $N_S$ ,  $N_D$ ,  $N_V$ ,  $N_P$ . Critical exponents  $\theta_i$  are minus the eigenvalues of the stability matrix  $\partial\beta_i/\partial g_j$ , so  $\text{Re } \theta_i > 0$  means UV-relevant.

## 3 The derived Einstein–Hilbert flow with matter

### 3.1 Inputs [INPUT]

Decompose  $h_{\mu\nu} = \hat{h}_{\mu\nu} + \frac{1}{4}\bar{g}_{\mu\nu}\varphi$  with  $\hat{h}$  traceless. In the gauge of Assumption 2.1 the quadratic form on  $S^4$  is diagonal [3] (Eqs. (4.12), (4.13), (4.17)–(4.19) of that reference, which we checked in the arXiv text):

$$\hat{h} : -\bar{D}^2 - 2\bar{\Lambda}_k + C_T R, \quad \varphi : -\bar{D}^2 - 2\bar{\Lambda}_k + C_S R, \quad \text{ghosts: } -\bar{D}^2 - \frac{1}{4}R, \quad (2)$$

with  $C_T = \frac{d(d-3)+4}{d(d-1)} = \frac{2}{3}$ ,  $C_S = \frac{d-4}{d} = 0$  for  $d = 4$ . The overall sign of the  $\varphi$  operator (the conformal-factor problem) cancels in the ratio entering the Wetterich equation if the cutoff carries the same factor. The heat-kernel coefficient  $\frac{1}{6}R$  per component for Bochner Laplacians is standard [9]. The Wetterich equation is Ref. [2]. These are the only inputs that are not recomputed.

### 3.2 The trace and the flow equations [DERIVED]

The ten metric components have weight  $+\frac{1}{2}$  each and mass  $w = -2\lambda$ ; the four ghost components have weight  $-1$  and mass 0. With threshold functions  $\Phi_n^p(w)$ ,  $\tilde{\Phi}_n^p(w)$  built from the profile  $R^{(0)}$ , expanding to first order in  $R/k^2$  at fixed  $\eta$  gives  $\partial_t \Gamma_k = \int \sqrt{\bar{g}} \{k^4 a_0 + k^2 R a_1\}$  with

$$16\pi^2 a_0 = 5[2\Phi_2^1(w) - \eta\tilde{\Phi}_2^1(w)] - 8\Phi_2^1(0), \quad (3)$$

$$16\pi^2 a_1 = \frac{5}{3}\Phi_1^1(w) - \frac{5}{6}\eta\tilde{\Phi}_1^1(w) - 6\Phi_2^2(w) + 3\eta\tilde{\Phi}_2^2(w) - \frac{4}{3}\Phi_1^1(0) - 2\Phi_2^2(0). \quad (4)$$

Comparing with  $\partial_t \Gamma_k$  evaluated on the EH ansatz (the coefficient of  $R$  gives  $\eta$ , the volume term gives  $\beta_\lambda$ ):

$$\eta = 16\pi\tilde{G}a_1, \quad \beta_\lambda = -(2 - \eta)\lambda + 8\pi\tilde{G}a_0, \quad \beta_{\tilde{G}} = (2 + \eta)\tilde{G}. \quad (5)$$

Since  $a_1$  is linear in  $\eta$ ,  $\eta = \tilde{G}B_1/(1 - \tilde{G}B_2)$ . For the Litim profile  $\Phi_n^p(w) = 1/[\Gamma(n+1)(1+w)^p]$  and  $\tilde{\Phi}_n^p(w) = 1/[\Gamma(n+2)(1+w)^p]$ , so

$$B_1 = \frac{1}{3\pi} \left[ \frac{5}{u} - \frac{9}{u^2} - 7 \right], \quad B_2 = -\frac{1}{12\pi} \left[ \frac{5}{u} - \frac{6}{u^2} \right], \quad 8\pi\tilde{G}a_0 = \frac{\tilde{G}}{2\pi} \left[ \frac{5}{u} - 4 - \frac{5\eta}{6u} \right]. \quad (6)$$

The anomalous dimension is the coefficient of  $R$ ; it cannot be read off from the volume term, which determines only  $\partial_t \bar{\Lambda}_k$ .

**Checks.** The script `eh_flow_qdg.py` assembles  $B_1, B_2$  and the volume term from the mode table and the threshold functions and compares them with Eq. (6): the agreement is at the level of  $10^{-16}$  at four values of  $\lambda$ . The closed forms of the Litim threshold functions agree with numerical quadrature of their defining integrals to  $3 \times 10^{-17}$ . For pure gravity the system has a single admissible fixed point,

$$(\tilde{G}_*, \lambda_*) = (0.7073, 0.1932), \quad \theta = 1.475 \pm 3.043i, \quad \tilde{G}_* \lambda_* = 0.137. \quad (7)$$

Lauscher and Reuter report, for the pure EH truncation with type A and type B cutoffs and  $\alpha = 1$ , a complex pair with  $1.4 \lesssim \theta' \lesssim 1.8$  and  $2.3 \lesssim \theta'' \lesssim 4$  and  $\tilde{G}_* \lambda_* \approx 0.12$  (Section 5.3.1 of Ref. [4]); our values lie in these ranges and have the same size of  $\tilde{G}_* \lambda_*$ . We have not compared individual digits with a published Litim-cutoff table.

### 3.3 Minimally coupled matter [DERIVED]

For a field with fluctuation operator  $\Delta = -\bar{D}^2 + E$  on a bundle of fibre dimension  $N$  the heat-kernel coefficient on  $S^4$  is  $\text{tr} b_2 = \frac{R}{6}N - \text{tr} E$ . We use a cutoff that is a function of the full operator  $\Delta$  (“type B”), so that  $E$  is absorbed in the argument of the threshold functions. Each field contributes  $\partial_t \Gamma_k \ni \int \sqrt{g} \{k^4 Y \Phi_2^1(0)/(16\pi^2) + k^2 R X \Phi_1^1(0)/(96\pi^2)\}$ , normalised so that a real minimally coupled scalar has  $X = Y = 1$ . Matching as in Eq. (5) gives, for the Litim cutoff,

$$\Delta\eta = \frac{\tilde{G}X}{6\pi}, \quad \Delta\beta_\lambda = \frac{\tilde{G}Y}{4\pi}, \quad X = N_S + 2N_D - 4N_V - 3N_P, \quad Y = N_S - 4N_D + 2N_V + 3N_P. \quad (8)$$

The coefficients come from: Dirac fermion,  $\not{D}^2 = -\bar{D}^2 + \frac{R}{4}$ ,  $N = 4$ , weight  $-\frac{1}{2}$ , giving  $X_D = 2$ ,  $Y_D = -4$ ; massless vector in Feynman gauge,  $E = R^\mu{}_\nu$  (so  $\text{tr} E = R$ ) together with two scalar ghosts of weight  $-1$ , giving  $X_V = -4$ ,  $Y_V = 2$ ; Proca field in the massless limit (Assumption 2.2) = vector + scalar, giving  $X_P = -3$ ,  $Y_P = 3$ . The same combinations  $N_S + 2N_D - 4N_V$  and  $N_S - 4N_D + 2N_V$  appear with prefactors  $\tilde{G}^2/(6\pi)$  in  $\beta_{\tilde{G}}$  and  $\tilde{G}/(4\pi)$  in  $\beta_{\tilde{\lambda}}$  in the one-loop flow of Ref. [5] (their Eqs. (35)–(36), type II cutoff,  $\alpha = \rho = 1$ , which we checked in the arXiv text); we have not compared the finite- $\eta$  terms. *The value of  $X$  depends on the matter-cutoff type:* with a cutoff that is a function of  $-\bar{D}^2$  alone,  $E$  stays in the propagator, the same counting gives  $X_D = -1$ ,  $X_V = -1$  and  $X_P = 0$ , and  $Y$  is unchanged (script `sensitivity_xy.py`). Matter enters the numerator of  $\eta$  but not the denominator  $1 - \tilde{G}B_2$ .

For  $n_f$  Dirac fermions and two Proca fields,

$$X = 2n_f - 6, \quad Y = 6 - 4n_f. \quad (9)$$

If the two vectors are instead counted as massless,  $X = 2n_f - 8$ ,  $Y = 4 - 4n_f$ . The two coefficients must be taken from the same counting.

### 3.4 The system [DERIVED]

$$\beta_{\tilde{G}} = (2 + \eta)\tilde{G}, \quad \eta = \frac{\tilde{G}[B_1(\lambda) + X/(6\pi)]}{1 - \tilde{G}B_2(\lambda)}, \quad \beta_\lambda = -(2 - \eta)\lambda + \frac{\tilde{G}}{2\pi} \left[ \frac{5}{u} - 4 - \frac{5\eta}{6u} + \frac{Y}{2} \right]. \quad (10)$$

A non-Gaussian fixed point is admissible if  $\tilde{G}_* > 0$ ,  $\lambda_* < \frac{1}{2}$  ( $u > 0$ ) and  $1 - \tilde{G}_* B_2(\lambda_*) > 0$ , so that it lies on the same side of the pole of  $\eta$  as the Gaussian fixed point.

**Proposition 3.1** (Closed form of the fixed points). *The admissible non-Gaussian fixed points of Eq. (10) are*

$$\lambda_* = \frac{1 - u}{2}, \quad \tilde{G}_* = \frac{2\pi}{N(u)}, \quad N(u) = \frac{4}{u^2} - \frac{5}{2u} + \frac{c}{6}, \quad c = 14 - X, \quad (11)$$

where  $u$  is a root of

$$2cu^3 - (2c + 54 - 3Y)u^2 + 118u - 48 = 0 \quad (12)$$

with  $u > 0$ ,  $N(u) > 0$  and  $N(u) - 2\pi B_2(u) = \frac{3}{u^2} - \frac{5}{3u} + \frac{c}{6} > 0$ .

*Proof.* For  $\tilde{G} \neq 0$ ,  $\beta_{\tilde{G}} = 0$  means  $\eta = -2$ . Inserting this in the expression for  $\eta$  gives  $\tilde{G}[B_1 + X/(6\pi) - 2B_2] = -2$ , i.e.  $\tilde{G} = 2\pi/N(u)$  with  $N = -\pi B_1 + 2\pi B_2 - X/6$ ; with Eq. (6),  $-\pi B_1 = -\frac{5}{3u} + \frac{3}{u^2} + \frac{7}{3}$  and  $2\pi B_2 = -\frac{5}{6u} + \frac{1}{u^2}$ , which gives  $N(u)$ . With  $\eta = -2$  the condition  $\beta_\lambda = 0$  reads  $4\lambda = \frac{\tilde{G}}{2\pi}[\frac{20}{3u} - 4 + \frac{Y}{2}]$ . Substituting  $\tilde{G}$ , multiplying by  $6u^2N(u)$  and using  $\lambda = (1 - u)/2$  gives  $2(1 - u)(24 - 15u + (14 - X)u^2) = u(40 + (3Y - 24)u)$ , which is Eq. (12). Conversely each root satisfying the inequalities gives a fixed point; the last inequality is  $1 - \tilde{G}_*B_2 = 1 - 2\pi B_2/N > 0$  with  $N > 0$ .  $\square$

For the QDG-like content, Eq. (9) gives

$$(40 - 4n_f)u^3 - (76 + 8n_f)u^2 + 118u - 48 = 0. \quad (13)$$

At  $n_f = 3$  this is  $28u^3 - 100u^2 + 118u - 48 = 0$ , whose only real root is  $u = 1.6358$  (the other two roots are a complex pair). At that root  $\eta = -2.000000000000$  and  $\beta_\lambda = 9 \times 10^{-16}$  when the full flow, Eq. (10), is evaluated. As an independent check on the cubic, 2000 random starts of a Newton solver on Eq. (10) (with  $\tilde{G} \in [0.03, 100]$ ,  $\lambda \in [-40, 0.45]$ ) converge only to this one admissible zero.

## 4 The fixed point for QDG-like content

### 4.1 Results [DERIVED]

Table 1 gives the fixed point for  $n_f$  Dirac fermions and two Proca fields (Litim cutoff,  $a = 1$ ).

Table 1: Admissible fixed point of Eq. (10) for  $n_f$  Dirac fermions and two Proca fields in the massless limit,  $(X, Y) = (2n_f - 6, 6 - 4n_f)$ . Script `eh_flow_qdg.py`; the residual  $\max |\beta_i|$  is below  $10^{-12}$  in every row, and  $1 - \tilde{G}_*B_2$  lies between 0.85 and 1.11.

| $n_f$ | $X$ | $Y$ | $\tilde{G}_*$ | $\lambda_*$ | $\tilde{G}_*\lambda_*$ | $\theta_{1,2}$      |
|-------|-----|-----|---------------|-------------|------------------------|---------------------|
| 0     | -6  | 6   | 0.5093        | 0.2290      | 0.117                  | $2.171 \pm 3.263 i$ |
| 1     | -4  | 2   | 0.6204        | 0.2030      | 0.126                  | $1.760 \pm 3.023 i$ |
| 2     | -2  | -2  | 0.8362        | 0.1568      | 0.131                  | $1.226 \pm 2.531 i$ |
| 3     | 0   | -6  | 2.7319        | -0.3179     | -0.868                 | 2.207, 0.723        |
| 4     | 2   | -10 | 3.9037        | -1.0676     | -4.17                  | 1.797, 3.161        |
| 6     | 6   | -18 | 5.9848        | -2.8594     | -17.1                  | 2.146, 3.695        |
| 8     | 10  | -26 | 11.836        | -7.8175     | -92.5                  | 2.249, 3.899        |
| 9.5   | 13  | -32 | 46.841        | -37.110     | -1738                  | 2.286, 3.980        |

The branch is continuous in  $n_f$ .  $\lambda_*$  changes sign at  $n_f = 17/6 \simeq 2.83$  (for two massless vectors, at  $n_f = 7/3$ ), and the pair of exponents turns from complex to real at  $n_f \simeq 2.873$  (the discriminant of the stability matrix vanishes there). The point  $n_f = 3$  is therefore just beyond a qualitative change: for  $n_f \leq 2$  the fixed point is the familiar EH one with  $\lambda_* > 0$  and complex exponents, and for  $n_f \geq 3$  it has two real relevant directions.

**Proposition 4.1** (End of the branch). *Let  $Y < 8$ . As  $c = 14 - X \rightarrow 0^+$  one root of Eq. (12) escapes to infinity,  $u \simeq (54 - 3Y)/(2c)$ , and satisfies the three conditions of Proposition 3.1, with  $\lambda_* \simeq -(54 - 3Y)/(4c) \rightarrow -\infty$  and  $\tilde{G}_* \simeq 2\pi/\{c[\frac{1}{6} - \frac{5}{54-3Y}]\} \rightarrow +\infty$ . For  $c = 0$  the cubic degenerates to  $(3Y - 54)u^2 + 118u - 48 = 0$ , which has no real root if and only if  $Y < 18 - 118^2/576 = -6.17\dots$*

*Proof.* For small  $c$  the leading terms of Eq. (12) are  $2cu^3 - (54 - 3Y)u^2$ , which gives  $u \simeq (54 - 3Y)/(2c)$ , positive for  $Y < 18$ . Then  $4/u^2 = O(c^2)$  and  $-5/(2u) \simeq -5c/(54 - 3Y)$ , so  $N \simeq c[\frac{1}{6} - \frac{5}{54-3Y}]$ , positive iff  $54 - 3Y > 30$ , i.e.  $Y < 8$ . The third condition reads  $N - 2\pi B_2 \simeq c[\frac{1}{6} - \frac{10}{3(54-3Y)}] > 0$ , which holds for  $54 - 3Y > 20$ , hence for  $Y < 8$ . The discriminant of the quadratic is  $118^2 - 192(54 - 3Y)$ .  $\square$

For the family (9),  $c = 0$  at  $n_f = 10$  (two Proca) and at  $n_f = 11$  (two massless vectors), and  $Y < -6.17$  there, so the fixed point runs off to  $\tilde{G}_*$ ,  $|\lambda_*| \rightarrow \infty$  at those values; close to the end,  $\lambda_* \simeq -19.5/(10 - n_f)$  and  $\tilde{G}_* \simeq (52\pi/7)/(10 - n_f)$  (checked at  $n_f = 9.9, 9.99, 9.999$ ). A scan of  $n_f$  from 10 to 200 in steps of 0.01 finds no admissible root for the Proca counting (from 11 for the massless counting); this is a numerical observation, not a consequence of Proposition 4.1. The value 10 is a feature of the type-B matter counting; with the type-A counting of Section 6 the branch does not end there. The statement “the fixed point exists only up to a maximal number of species” is in qualitative agreement with the abstract of Ref. [5], which states that for a given number of gauge fields there is a maximal number of scalar and fermion degrees of freedom compatible with a fixed point at positive Newton coupling.

## 4.2 Reliability at $n_f = 3$

The fixed point at  $n_f = 3$  has  $\tilde{G}_* = 2.73$  and  $\lambda_* = -0.318$ , with  $\tilde{G}_*\lambda_* = -0.87$ ; for pure gravity and for  $n_f \leq 2$  the product is  $\approx 0.12$ – $0.14$ . We call a fixed point *within the conventional range* if  $\tilde{G}_* < 2$  and  $\lambda_* > -1$ . These two numbers are a convention for where an expansion in  $R/k^2$  with higher operators neglected is plausible, not a derived bound. For the Litim cutoff with  $a = 1$  and type-B matter the  $n_f = 3$  point is just outside it, and it moves quickly outside for larger  $n_f$  ( $|\lambda_*| > 5$  for  $n_f \gtrsim 7.3$ ). Sections 5–6 show that the position is not robust under the regulator and the matter-cutoff convention. What can be said is that the truncation gives no reason to trust the numerical position of the  $n_f = 3$  point, and that nothing in this note tests the effect of higher-curvature operators for QDG-like content.

**Remark 4.2** (Standard Model content). For  $N_S = 4$ ,  $N_D = 22.5$ ,  $N_V = 12$  (minimal coupling only,  $X = 1$ ,  $Y = -62$ ) the same equations give  $(\tilde{G}_*, \lambda_*) = (3.217, -4.393)$  with  $\theta = (2.083, 3.926)$ . Adding the two QDG vectors as Proca fields to the Standard-Model fields ( $X = -5$ ,  $Y = -56$ ) gives  $(2.187, -2.693)$  with  $\theta = (2.044, 3.883)$  and an admissible fixed point for all nine regulators of Section 5 (script `sm_plus_qdg.py`). These are applications of Eq. (8), not QDG predictions: the identification of the QDG fermions with the Standard-Model fermions is not made here.

# 5 Regulator dependence

## 5.1 Nine regulators [DERIVED]

We repeated the calculation for the optimised profile  $R^{(0)} = a(1-z)\theta(1-z)$  with  $a = 0.5, 0.75, 1, 1.5, 2, 3$  and the exponential profile  $R^{(0)} = az/(e^z - 1)$  with  $a = 0.5, 1, 2$ , computing the threshold functions by Gauss–Legendre quadrature and scanning  $\lambda \in [-80, \lambda_{\max}]$ . We call a fixed point *admissible* if in addition  $\text{Re } \theta_i > 0$  for both exponents, and  $\rho$  the fraction of the nine regulators for which an admissible fixed point exists (the regulator-robust-existence criterion of the second edition of Ref. [13], which is a necessary condition only: it says nothing about gauge, truncation, or the position of the point).

1. **Existence is robust.** For the Proca counting  $\rho = 1$  for every  $n_f \leq 9$  on a grid of step 0.5,  $\rho = 0.78$  at 9.5, 0.22 at 10 and 0 for  $n_f \geq 10.5$ ; for two massless vectors the corresponding values are  $\rho = 1$  for  $n_f \leq 10$ , 0.78 at 10.5, 0.22 at 11. At  $n_f = 3$  all nine regulators have an admissible fixed point.

Table 2: The fixed point closest to  $\lambda = 0$  at  $n_f = 3$  (two Proca,  $X = 0$ ,  $Y = -6$ ) for nine regulators. Script `rre_qdg.py`.

| profile     | $a$  | $\tilde{G}_*$ | $\lambda_*$ | $\theta_{1,2}$      |
|-------------|------|---------------|-------------|---------------------|
| optimised   | 0.5  | 2.7888        | -0.1206     | 1.692, 0.280        |
| optimised   | 0.75 | 2.8339        | -0.2426     | 2.089, 0.555        |
| optimised   | 1    | 2.7319        | -0.3179     | 2.207, 0.723        |
| optimised   | 1.5  | 2.5703        | -0.4266     | 2.310, 0.926        |
| optimised   | 2    | 2.4659        | -0.5090     | 2.359, 1.053        |
| optimised   | 3    | 2.3434        | -0.6354     | 1.214, 2.410        |
| exponential | 0.5  | 0.8067        | 0.0773      | $0.091 \pm 1.856 i$ |
| exponential | 1    | 0.6512        | 0.0578      | $0.407 \pm 1.391 i$ |
| exponential | 2    | 0.5804        | -0.0714     | $0.600 \pm 0.718 i$ |

- Position and character are not.** At  $n_f = 3$  the position  $\tilde{G}_*$  ranges from 0.58 to 2.83. For the six optimised regulators  $\tilde{G}_* > 2.3$  and the exponents are real,  $\lambda_* \in [-0.64, -0.12]$ ; for the three exponential regulators  $\tilde{G}_* < 0.81$  and the exponents are a complex pair with small real part. Only the exponential regulators put the point inside the conventional range ( $\tilde{G}_* < 2$ ,  $\lambda_* > -1$  holds for three of the nine regulators, and  $\lambda_* > -1$  for all nine). The value of  $n_f$  at which the exponents become real (grid step 0.1) is 2.8–3.0 for the optimised regulators and 3.1–3.2 for the exponential ones, so  $n_f = 3$  lies on different sides of the change of character depending on the regulator.
- The farther the larger.** For  $n_f = 4, 6, 8$  the maximum of  $\tilde{G}_*$  over the nine regulators is 4.95, 7.54, 13.9 and the minimum of  $\lambda_*$  is -3.0, -8.3, -21.2, so the large- $n_f$  part of the branch is far from any region where the truncation can be trusted.

Lauscher and Reuter note that the existence of a fixed point is a scheme-independent feature in the exact theory, while its precise location and, in a truncation, the exponents are not [4]. The nine-regulator result is the first of these statements applied to a small family; it is not a proof for the full theory.

## 6 Sensitivity to the omitted sector and to the matter-cutoff type

The flow depends on the matter content only through  $(X, Y)$ . The omitted ingredients of QDG (non-minimal couplings of the Stueckelberg scalar and of the vectors to curvature, the dilaton–Weyl-vector coupling, the axial coupling to fermions, finite masses) change  $X$  and  $Y$  by amounts that we cannot compute here. We therefore ask how far  $(X, Y)$  can move away from  $(0, -6)$  before the  $n_f = 3$  fixed point is lost, and how its character changes (script `sensitivity_xy.py`; Litim cutoff,  $a = 1$ , exact cubic). This is a sensitivity analysis, not a computation of the omitted sector.

Three conclusions follow.

- Existence* is stable. Along  $Y = -6$  an admissible fixed point exists for every  $X \in [-100, 277.35]$  (window searched), and along  $X = 0$  for every  $Y \in [-100, 100]$ . Shifts of order one in  $X$  or  $Y$  do not remove it.
- Position and character* are not stable. A shift of +1 in  $Y$  moves the point from  $(2.73, -0.32)$  to  $(1.39, 0.05)$  and turns the exponents back into a complex pair; a shift of -3 in  $X$  moves it inside the conventional range.

Table 3: The fixed point of Eq. (10) for shifted contents  $(X, Y) = (0, -6) + (\delta X, \delta Y)$ . A scalar with non-minimal coupling  $\xi R$  has  $X = 1 - 6\xi$  (type-B counting,  $E = \xi R$ ) and the same  $Y$ , so  $\delta X = -1$  corresponds to  $\xi = 1/6$  for one scalar.

| $\delta X$ | $\delta Y$ | $\tilde{G}_*$ | $\lambda_*$ | $\theta_{1,2}$      |
|------------|------------|---------------|-------------|---------------------|
| 0          | 0          | 2.7319        | -0.3179     | 2.207, 0.723        |
| -1         | 0          | 2.3341        | -0.2204     | 1.997, 0.629        |
| -2         | 0          | 2.0279        | -0.1528     | 1.764, 0.709        |
| -3         | 0          | 1.8188        | -0.1132     | 1.491, 0.978        |
| +2         | 0          | 3.6833        | -0.5688     | 1.131, 2.514        |
| 0          | +1         | 1.3892        | 0.0518      | $0.656 \pm 1.363 i$ |
| 0          | -1         | 3.0159        | -0.5000     | 1.206, 2.579        |
| 0          | +2         | 1.0868        | 0.1130      | $0.820 \pm 2.055 i$ |
| 0          | -2         | 3.1232        | -0.6264     | 1.437, 2.791        |
| +2         | +2         | 1.0875        | 0.1250      | $0.697 \pm 2.228 i$ |

3. *The matter-cutoff type* is such a shift. With the type-A counting ( $X_D = X_V = -1$ ,  $X_P = 0$ , so  $X = -n_f$  for the QDG-like content,  $Y$  unchanged; verified in `sensitivity_xy.py` from the mode table) the  $n_f = 3$  point is  $(\tilde{G}_*, \lambda_*) = (1.819, -0.113)$  with real exponents (1.491, 0.978), inside the conventional range, and for  $n_f = 4, 6, 10, 20, 40$  the fixed point stays at  $\tilde{G}_* = 2.30, 2.13, 1.74, 1.18, 0.72$  with  $\lambda_* = -0.53, -0.90, -1.33, -1.86, -2.30$ ; there is no end of the branch at  $n_f = 10$  (the branch continues at least to  $n_f = 40$ ). The type-B convention is the one that reproduces the combinations of Ref. [5] and is used throughout; the point is that the result “ $n_{\text{crit}} = 10$ ” belongs to that convention.

## 7 The model flow of the first version

### 7.1 Definition and status [MODEL]

The first version used (with  $A(\lambda) = \frac{5}{1-2\lambda} - \frac{3}{1-\lambda} - 1$ ,  $B(\lambda) = A(\lambda) + 1$ )

$$\eta = -\frac{\tilde{G}A(\lambda)}{1 - \tilde{G}B(\lambda)} + C_\eta \tilde{G}, \quad \beta_{\tilde{G}} = (2 + \eta)\tilde{G}, \quad (14)$$

$$\beta_\lambda = -(2 - \eta)\lambda + \frac{\tilde{G}}{2\pi}B(\lambda) + C_{\text{bl}}\tilde{G} + \frac{72\lambda^2}{16\pi^2}, \quad C_\eta = \frac{n_f - 4}{3\pi}. \quad (15)$$

This is a model, for the following reasons (script `model_flow_check.py`). (i)  $A$  and  $B$  have the form of signed multiplicities of the volume term ( $5 = \frac{9}{2} + \frac{1}{2}$  metric components at mass  $-2\lambda$ ;  $-\frac{3}{1-\lambda} - 1$  standing for ghost-like contributions), but they are used as the coefficient of  $R$ ; Section 3 shows that the coefficient of  $R$  is  $B_1, B_2$  of Eq. (6). (ii) The model has poles at  $\lambda = \frac{1}{2}$  and  $\lambda = 1$ ; the derived flow has only  $\lambda = \frac{1}{2}$ . (iii) The term  $72\lambda^2/(16\pi^2)$  is not derived from the action. (iv) The pure-gravity limit of the model ( $C_\eta = C_{\text{bl}} = 0$ ) is  $(\tilde{G}_*, \lambda_*) = (0.3502, 0.0313)$ , not the EH fixed point (0.7073, 0.1932). (v)  $C_\eta$  corresponds to two massless vectors ( $X = 2n_f - 8$ ) while the consistent coefficient  $C_{\text{bl}}$  below corresponds to two Proca fields ( $Y = 6 - 4n_f$ ). The model is not a gauge or parametrisation variant of the EH flow of Section 3: the pole at  $\lambda = 1$  is absent from the latter. Whether it can be obtained from another gauge or from the distortion sector is not known.

### 7.2 The coefficient $C_{\text{bl}}$ and the claim of absence at $n_f = 3$ [DERIVED]

By the matching used in Eq. (8), a matter term  $\Delta\beta_\lambda = \tilde{G}Y/(4\pi)$  corresponds to  $C_{\text{bl}} = Y/(4\pi)$ . For  $n_f$  Dirac fermions and two Proca fields  $Y = 6 - 4n_f$ , i.e.  $C_{\text{bl}} = (3 - 2n_f)/(2\pi)$ . The

first version used  $C_{\text{bl}} = 2(1 - n_f)/\pi$ , which corresponds to  $Y = 8(1 - n_f)$ : the contribution per fermion is  $-8$  instead of  $-4$ , and the constant does not match either the Proca (6) or the massless-vector (4) counting. Table 4 shows the consequence.

Table 4: Fixed points of the model flow, Eqs. (14)–(15) (physical branch:  $\tilde{G}_* > 0$ ,  $\lambda_* < \frac{1}{2}$ ,  $1 - \tilde{G}_*B > 0$ , smallest  $|\lambda_*|$ ), with the coefficient of the first version and with the consistent one. Script `model_flow_check.py`.

| $n_f$ | $C_{\text{bl}} = 2(1 - n_f)/\pi$ |                | $C_{\text{bl}} = (3 - 2n_f)/(2\pi)$ |                |
|-------|----------------------------------|----------------|-------------------------------------|----------------|
|       | $(\tilde{G}_*, \lambda_*)$       | $\theta_{1,2}$ | $(\tilde{G}_*, \lambda_*)$          | $\theta_{1,2}$ |
| 0     | (0.2839, 0.0757)                 | 10.06, 3.66    | (0.2964, 0.0659)                    | 9.86, 3.65     |
| 1     | (0.3464, 0.0309)                 | 9.50, 3.61     | (0.3278, 0.0438)                    | 9.71, 3.62     |
| 2     | (0.4727, $-0.0425$ )             | 8.86, 3.46     | (0.3699, 0.0165)                    | 9.57, 3.59     |
| 2.6   | (0.7166, $-0.1442$ )             | 8.84, 2.91     | —                                   | —              |
| 3     | none                             | —              | (0.4318, $-0.0194$ )                | 9.45, 3.52     |
| 4     | none                             | —              | (0.5411, $-0.0733$ )                | 9.45, 3.37     |

With the consistent coefficient the model has a fixed point at  $n_f = 3$ ,  $(\tilde{G}_*, \lambda_*) = (0.4318, -0.0194)$ ,  $\theta = (9.446, 3.519)$ ; there is a second solution on the unphysical side of the pole ( $1 - \tilde{G}B < 0$ ) at  $(5.149, -0.481)$  and a third at large coupling,  $(362.5, -15.66)$ , with complex exponents. The physical root is lost only when  $1 - \tilde{G}_*B(\lambda_*) \rightarrow 0$ . That happens at  $(\tilde{G}, \lambda) = (1, (2 - \sqrt{6})/2)$  where  $A = 0$  and  $B = 1$ ; with  $\eta = -2$  there,  $\beta_\lambda = 0$  gives  $C_{\text{bl}}(n_{\text{crit}}) = 4\lambda_c - \frac{1}{2\pi} - \frac{72\lambda_c^2}{16\pi^2}$  with  $\lambda_c = (2 - \sqrt{6})/2$ , i.e.

$$n_{\text{crit}}^{\text{model}} = 4.896577961599 \dots \quad (16)$$

Even with the coefficient of the first version the physical root disappears at  $n_f \simeq 2.76$ , not between  $n_f = 2$  and  $n_f = 3$  as a statement about integer fermion number: “ $n_f \leq 2$  yes,  $n_f = 3$  no” reflects a non-integer threshold ( $n_f \simeq 2.76$ ) of an inconsistent model. We conclude that the claim of the first version, that no non-trivial fixed point exists at  $n_f = 3$ , is not supported, neither in the derived flow (Section 4) nor in the model flow with consistent coefficients. The residual statements of the first version that remain true are those about the numbers of the model with its own coefficients (Table 4, left); they have no bearing on QDG.

## 8 Discussion

### 8.1 What the note establishes and what it does not

The derived flow is the flow of the Einstein–Hilbert sector with minimally coupled matter. Two statements about QDG require an extra hypothesis and are not results of this note: that the two QDG vectors enter as minimally coupled massless Proca fields (Assumption 2.2), and that the omitted operators do not alter the flow. Gauge and parametrisation dependence were not studied; only  $\alpha = 1$  and the linear split were used. Because  $\lambda_*$  and  $\tilde{G}_*$  are not gauge invariant, the numbers above are statements about this gauge.

**Remark 8.1** (Gauge, Yukawa and mass parameters). The first version listed beta functions for  $g_V$ ,  $g_a$  and  $y$ . They are not part of the present flow, and we make no claim about them. The gravitational contribution to the running of gauge couplings has been found to depend on the gauge [8]; any such statement for  $g_V, g_a$  would have to be derived in the QDG framework with the matter-cutoff and gauge dependence under control. The scale-generation discussion of the first version, which was evaluated at a point that was not a fixed point, is not reinstated.

Table 5: Status of the results of this note.

| Result                                                                | Status              | Comment                                                                  |
|-----------------------------------------------------------------------|---------------------|--------------------------------------------------------------------------|
| EH Hessian on $S^4$ , harmonic gauge                                  | INPUT               | Ref. [3], not rederived                                                  |
| Trace, $B_1$ , $B_2$ , flow Eq. (10)                                  | DERIVED             | Litim cutoff, $\alpha = 1$ , linear split, $S^4$                         |
| Matter coefficients $X$ , $Y$                                         | DERIVED             | type-B cutoff; combinations agree with Ref. [5]; $X$ depends on the type |
| Cubic for the fixed point, Proposition 3.1                            | PROVEN              | given Eq. (10)                                                           |
| Fixed point at $n_f = 3$ : $(2.73, -0.318)$ , $\theta = (2.21, 0.72)$ | DERIVED             | Litim $a = 1$ , type B                                                   |
| Existence for $n_f < 10$ , nine regulators                            | DERIVED (numerical) | necessary condition only                                                 |
| Position, exponents, reliability at $n_f = 3$                         | NOT ESTABLISHED     | regulator and matter-cutoff dependent (Sections 5–6)                     |
| Model flow of the first version                                       | MODEL               | not derived from EH; absence at $n_f = 3$ withdrawn                      |
| Asymptotic safety of QDG                                              | OPEN                | Weyl-gauged sector not in the flow                                       |

## 8.2 Open problems

**Open Problem 8.2** (Weyl-gauged sector). Compute the contribution to  $\eta$  and  $\beta_\lambda$  of the dilaton, the Weyl vector  $V_\mu$  and the axial vector  $\tilde{a}_\mu$  with their non-minimal couplings and masses, instead of the minimally coupled massless Proca counting. Table 3 gives the size of the shifts in  $(X, Y)$  that would matter.

**Open Problem 8.3** (Truncation). Repeat the analysis with higher-curvature operators. The  $R^2$  truncation of Ref. [4] exists for pure gravity, and the regulator-robust-existence analysis of Ref. [13] extends it to Standard-Model matter; neither has been done for the QDG-like content at  $n_f = 3$ .

**Open Problem 8.4** (Gauge and parametrisation). Determine the dependence on  $\alpha$  (including the physical limit  $\alpha \rightarrow 0$ ) and on an exponential metric parametrisation. Every statement above is for  $\alpha = 1$  and the linear split.

**Open Problem 8.5** (Matter cutoff). Fix the matter-cutoff convention by a principle rather than by agreement with a reference, and quantify the type-A/type-B difference of Section 6 for gravity coupled to Proca fields. A first study of asymptotically safe generalised Proca theories with the functional renormalization group, with a different aim and a different set of couplings, is Ref. [7].

## 9 Conclusions

The claim of the first version of this note, that QDG has no non-Gaussian fixed point for  $n_f = 3$  Dirac fermions, is withdrawn. It rested on a phenomenological flow that was not derived from the Einstein–Hilbert action and, inside that model, on a matter coefficient twice too large per fermion. In the derived Einstein–Hilbert flow with minimally coupled matter of QDG-like content, a non-Gaussian fixed point exists for all  $n_f < 10$  and, at  $n_f = 3$ , at  $(\tilde{G}_*, \lambda_*) = (2.73, -0.318)$  with real exponents  $(2.21, 0.72)$  for the optimised cutoff; it exists for

nine of nine regulators, but its position ( $\tilde{G}_*$  from 0.58 to 2.83), the character of its exponents and its location relative to the conventional range depend on the regulator and on the matter-cutoff type. This is an existence statement in a truncation that omits the Weyl-gauged sector of QDG. It is neither a proof of asymptotic safety for QDG nor an argument against it.

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## A Scripts and core code

Every number computed in this paper is reproduced by one of the following scripts (NumPy and SciPy only; the outputs are stored next to the scripts as `*.out`).

- `eh_flow_qdg.py`:  $B_1, B_2$  from the mode table, flow, cubic, Table 1, the end of the branch, pure gravity, Standard-Model content, multi-start check.
- `rre_qdg.py` (with `rre.py`, `rre_run50.py`): nine regulators, Table 2,  $\rho(n_f)$ .
- `sensitivity_xy.py`, `sm_plus_qdg.py`: Table 3, type-A counting, Standard-Model content.
- `model_flow_check.py`: Section 7, Table 4,  $n_{\text{crit}}^{\text{model}}$ .

The core of the derived flow and of the fixed-point search is:

```
import numpy as np
pi = np.pi

def B1B2(lam):
    # B1, B2 of Eq. (6), Litim cutoff, u = 1-2*lambda
    u = 1 - 2*lam
    return (5/u - 9/u**2 - 7)/(3*pi), -(5/u - 6/u**2)/(12*pi)

def eta(G, lam, X):
    B1, B2 = B1B2(lam)
    return G*(B1 + X/(6*pi))/(1 - G*B2)

def beta_G(G, lam, X, Y):
    return (2 + eta(G, lam, X))*G

def beta_lam(G, lam, X, Y):
    u = 1 - 2*lam; e = eta(G, lam, X)
    return -(2 - e)*lam + G/(2*pi)*(5/u - 4 - 5*e/(6*u) + Y/2)

def fixed_points(X, Y):
    # Proposition 3.1: cubic for u = 1-2*lambda
    c = 14 - X
    roots = np.roots([2*c, -(2*c + 54 - 3*Y), 118., -48.])
    out = []
    for u in roots:
        if abs(u.imag) < 1e-9 and u.real > 0:
            u = u.real; N = 4/u**2 - 2.5/u + c/6
            if N > 0:
                G = 2*pi/N; lam = (1 - u)/2
                if 1 - G*B1B2(lam)[1] > 0: out.append((G, lam))
    return out
```

```
nf = 3
print(fixed_points(2*nf - 6, 6 - 4*nf)) # [(2.7319, -0.3179)]
```

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