

Batalin–Vilkovisky Functional Renormalization Group for Quantum Distortion Gravity: Regulator-Induced Symmetry Breaking, Anomaly Status and Fixed-Point Existence

L. E. Pavese^{*} ¹

¹Independent Researcher, Italy

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Abstract

Quantum Distortion Gravity (QDG) is a metric-affine theory with gauged Weyl symmetry. We set up its Batalin–Vilkovisky (BV) formulation and combine it with the Wetterich functional renormalization group. The BRST transformations of $\text{Diff} \ltimes \text{Weyl}$ (with local Lorentz transformations for fermions) are written out, their nilpotency and the classical master equation are established, and the Faddeev–Popov action in unitary gauge is shown to have a field-independent Weyl-ghost determinant. For a regulator that does not respect the BRST symmetry we derive the modified Slavnov–Taylor identity with an explicit regulator term $\mathcal{K}_k$ and show that the Wetterich flow preserves it; the resulting “third equation” for $\partial_t \mathcal{K}_k$ is an integrability condition and a diagnostic of truncations, not an independent dynamical law. The 60 non-axial components of the Weyl-covariant distortion are auxiliary with constant propagator, so they form an ultralocal Gaussian block that contributes to no curvature-dependent coefficient; the axial vector is a Proca field and does contribute. The anomaly coefficients of the matter content are $(c, a) = (11/30, 53/120)$ for $n_f = 3$ Dirac fermions and two Proca fields; the graviton has no such pair, and the anomaly of the gauged Weyl symmetry is open. With the Einstein–Hilbert flow and minimally coupled matter, the QDG-like content has a non-Gaussian fixed point for every $n_f < 10$, regulator-robust up to $n_f \simeq 9$, but at $n_f = 3$ it lies at $g_* = 2.73$, $\lambda_* = -0.32$, outside the range where the truncation is reliable; Weyl-gauged operators, gauge dependence and non-minimal couplings are not included. QDG remains a proposal; the status of each result is stated throughout.

Keywords: quantum gravity, metric-affine gravity, Weyl symmetry, BRST–BV quantization, functional renormalization group, modified Slavnov–Taylor identity, Weyl anomaly, asymptotic safety

1 Introduction

Quantum Distortion Gravity (QDG) [1, 2] is a metric-affine theory in which the affine connection $\Gamma^\rho_{\mu\nu}$ is an independent field and local Weyl invariance is a gauge symmetry. The classical theory rests on the distortion tensor $D^\rho_{\mu\nu} = \Gamma^\rho_{\mu\nu} - \hat{\Gamma}^\rho_{\mu\nu}(g)$, on a no-go theorem for bare quadratic invariants of D , and on a Weyl-invariant action with a dilaton ϕ and a Weyl gauge vector V_μ .

This paper asks how the BRST–BV structure of this gauge theory interacts with the functional renormalization group (FRG), what is known about the Weyl anomaly in this setting, and what can be said about a non-Gaussian fixed point of the flow. Its results are the following.

^{*}Corresponding author e-mail: lucapv02@gmail.com

1. [DERIVED] The BRST transformations of $\text{Diff} \ltimes \text{Weyl} (\times \text{Lorentz})$, with the inhomogeneous term of $s\Gamma$ and the Lorentz ghost required for fermions, are nilpotent; S_{BV} satisfies the classical master equation, and in unitary gauge the Weyl ghosts decouple (Section 3).
2. [DERIVED] For a regulator that breaks the BRST symmetry the modified Slavnov–Taylor identity (mSTI) has an explicit regulator term $\mathcal{K}_k$; the Wetterich flow preserves it; differentiating gives a third equation, which is an integrability condition (Section 4).
3. [DERIVED] In the QDG action the 60 non-axial components of the Weyl-covariant distortion are auxiliary and ultralocal. The conformal-anomaly coefficients of the matter content are given; the graviton has none; the anomaly of the gauged symmetry is open (Section 5).
4. [DERIVED] / [ASSUMPTION] With the Einstein–Hilbert flow and minimally coupled matter, the QDG-like content has a regulator-robust non-Gaussian fixed point up to $n_f \simeq 9$, whose position and dependence on the field masses are computed; whether the Weyl-gauged sector modifies this is [OPEN] (Section 6).

Logical status. Every statement carries one of the tags [STANDARD] (standard result, source cited), [DERIVED] (derived in this paper, or reproduced by a script supplied with it), [ASSUMPTION] (an assumption stated as such), [MODEL] (a phenomenological model) or [OPEN] (open problem). *Theorem* and *Proposition* are proved under the stated hypotheses; *Remark* is not proved. The scripts (numpy only) are listed in Appendix A.

Conventions. Signature $(-, +, +, +)$, $\varepsilon^{0123} = +1$, $\nabla_\mu V^\rho = \partial_\mu V^\rho + \Gamma^\rho_{\mu\nu} V^\nu$, Weyl rescaling $g \rightarrow \Omega^2 g$, $\omega = \ln \Omega$, a field of weight w scales as Ω^w . Sections 2–3 and 5 are Lorentzian; Sections 4 and 6 are Euclidean (the FRG is formulated there), and Grassmann signs are suppressed in Section 4.

2 Classical foundations

2.1 Distortion tensor and its decomposition

With the Levi-Civita connection $\hat{\Gamma}$ of g , $D_{\rho\mu\nu} = g_{\rho\sigma} D^\sigma_{\mu\nu}$, torsion $T^\rho_{\mu\nu} = D^\rho_{\mu\nu} - D^\rho_{\nu\mu}$ and non-metricity $Q_{\mu\nu\rho} = -\nabla_\mu g_{\nu\rho} = D_{\rho\mu\nu} + D_{\nu\mu\rho}$. The curvature is $R^\rho_{\sigma\mu\nu} = \partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda} \Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda} \Gamma^\lambda_{\mu\sigma}$, so that $[\nabla_\mu, \nabla_\nu] V^\rho = R^\rho_{\sigma\mu\nu} V^\sigma$. Define the traces $A_\nu = D^\rho_{\rho\nu}$, $B_\mu = D^\rho_{\mu\rho}$, $C_\rho = D^\mu_{\rho\mu}$ and the axial vector $a^\sigma = \frac{1}{6} \varepsilon^{\sigma\rho\mu\nu} D_{\rho\mu\nu}$.

Proposition 2.1 (Irreducible decomposition). [DERIVED] Every $D_{\rho\mu\nu}$ is uniquely

$$D_{\rho\mu\nu} = \varepsilon_{\rho\mu\nu\sigma} a^\sigma + g_{\rho\mu} P_\nu + g_{\rho\nu} Q_\mu + g_{\mu\nu} R_\rho + \hat{q}_{\rho\mu\nu}, \quad (1)$$

with $P = \frac{1}{18}(5A - B - C)$, $Q = \frac{1}{18}(5B - A - C)$, $R = \frac{1}{18}(5C - A - B)$, and $\hat{q}$ traceless on all three pairs with vanishing totally antisymmetric part: $64 = 4 + 12 + 48$.

Proof. The totally antisymmetric part is $\varepsilon_{\rho\mu\nu\sigma} a^\sigma$, using $\varepsilon^{\tau\rho\mu\nu} \varepsilon_{\rho\mu\nu\sigma} = +6\delta^\tau_\sigma$ in Lorentzian signature. For $D^{(0)} = g_{\rho\mu} P_\nu + g_{\rho\nu} Q_\mu + g_{\mu\nu} R_\rho$ the traces are $(A, B, C)^\top = M(P, Q, R)^\top$ with $M = 3 \cdot \mathbb{K} + J$, J the all-ones matrix; $J^2 = 3J$ gives $M^{-1} = \frac{1}{3}(\mathbb{K} - \frac{1}{6}J)$, the stated P, Q, R . Set $\hat{q} = D - D^{(0)} - \varepsilon a$. $\square$

Three quadratic invariants are used: $I_1 = D_{\rho\mu\nu} D^{\rho\mu\nu}$, $I_2 = D_{\rho\mu\nu} D^{\mu\nu\rho}$ and $I_3 = C_\rho C^\rho$. The cyclic contraction in I_2 is the convention of the QDG papers; $D_{\rho\mu\nu} D^{\nu\rho\mu}$ gives the same invariant. In terms of the pieces of (1) [DERIVED]

$$\begin{aligned} I_1 &= -6a_\sigma a^\sigma + 4(P^2 + Q^2 + R^2) + 2(P \cdot Q + P \cdot R + Q \cdot R) + \hat{q}_{\rho\mu\nu} \hat{q}^{\rho\mu\nu}, \\ I_2 &= -6a_\sigma a^\sigma + (P^2 + Q^2 + R^2) + 5(P \cdot Q + P \cdot R + Q \cdot R) + \hat{q}_{\rho\mu\nu} \hat{q}^{\mu\nu\rho}, \\ I_3 &= (P + Q + 4R)_\rho (P + Q + 4R)^\rho, \end{aligned} \quad (2)$$

checked with random tensors on a random Lorentzian metric (`bvfrg_checks.py`); the axial vector is orthogonal to the other pieces in all three.

2.2 Weyl transformations and the no-go theorem

Under $g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}$ with Γ held fixed,

$$D^\rho{}_{\mu\nu} \rightarrow D^\rho{}_{\mu\nu} - I^\rho{}_{\mu\nu}, \quad I_{\rho\mu\nu} = g_{\rho\mu}v_\nu + g_{\rho\nu}v_\mu - g_{\mu\nu}v_\rho, \quad v = \partial\omega, \quad (3)$$

i.e. $(P, Q, R) \rightarrow (P, Q, R) - v(1, 1, -1)$ while a and $\hat{q}$ are inert. With $\phi \rightarrow \Omega^{-1}\phi$, $V_\mu \rightarrow V_\mu - \partial_\mu\omega$, put $W_{\rho\mu\nu} = g_{\rho\mu}V_\nu + g_{\rho\nu}V_\mu - g_{\mu\nu}V_\rho$ and $\tilde{D} = D - W$.

Proposition 2.2 (Weyl covariance). *[DERIVED] (i) $\tilde{D}^\rho{}_{\mu\nu}$ is invariant, $\tilde{D}_{\rho\mu\nu} \rightarrow \Omega^2 \tilde{D}_{\rho\mu\nu}$. (ii) The axial vector $\tilde{a}_\mu$ (index down) has weight 0. (iii) $D_\mu\phi \equiv (\partial_\mu - V_\mu)\phi \rightarrow \Omega^{-1}D_\mu\phi$. (iv) $\tilde{R} \equiv \hat{R} - 6\hat{\nabla}_\mu V^\mu - 6V_\mu V^\mu \rightarrow \Omega^{-2}\tilde{R}$. More generally $D_\mu\Phi = (\partial_\mu + wV_\mu)\Phi$ is covariant for a field of weight w .*

Proof. (i) follows from (3) and $V \rightarrow V - v$. (ii): $\tilde{a}^\sigma = \frac{1}{6}\varepsilon^{\sigma\rho\mu\nu}\tilde{D}_{\rho\mu\nu}$ has weight $-4 + 2 = -2$ and lowering the index adds 2. (iii) is immediate. (iv): with $\hat{R} \rightarrow \Omega^{-2}(\hat{R} - 6\Box\omega - 6(\partial\omega)^2)$ and $\hat{\nabla}'_\mu V'^\mu = \Omega^{-2}(\hat{\nabla}\cdot V' + 2\partial\omega\cdot V')$ in four dimensions, the terms in $\Box\omega$, $\partial\omega\cdot V$ and $(\partial\omega)^2$ cancel separately. These identities were also verified with exact jet arithmetic (`partI_II_audit_weyl.py`). $\square$

Theorem 2.3 (No-go for bare invariants). *[DERIVED] Let $S_{\text{bare}} = \int d^4x\sqrt{-g}\sum_i \alpha_i I_i[D]$ with constant α_i and Γ independent of g . It is invariant under all local Weyl rescalings if and only if $\alpha_1 = \alpha_2 = \alpha_3 = 0$. For infinitesimal ω*

$$\delta S_{\text{bare}} = \int d^4x\sqrt{-g}\left\{2\omega\sum_i \alpha_i I_i - [2\alpha_1(A + B - C) + 2\alpha_2 C - 4\alpha_3 C]^\rho \partial_\rho\omega\right\}. \quad (4)$$

Proof. Each $D_{\rho\mu\nu}$ has weight $+2$, each inverse metric -2 and $\sqrt{-g}$ weight 4 , so $\sqrt{-g}I_i$ has weight $+2$; this gives the first term of (4). The second term is the contraction of the shift $-I_{\rho\mu\nu}$ with D : $D^{\rho\mu\nu}I_{\rho\mu\nu} = (A + B - C)\cdot v$ gives $\delta_{\text{inh}}I_1 = -2(A + B - C)\cdot v$; for I_2 both factors vary, $\delta_{\text{inh}}I_2 = -2C\cdot v$; and $g^{\mu\nu}I_{\rho\mu\nu} = -2v_\rho$ gives $\delta_{\text{inh}}I_3 = 4C\cdot v$. For constant ω only the first term survives; taking $D = \chi(x)D_0$ with χ of compact support shows $\sum_i \alpha_i I_i[D_0] = 0$ for every constant D_0 , and I_1, I_2, I_3 are linearly independent (determinant 270 of the matrix of their values on a, P, R), so $\alpha_i = 0$. $\square$

Remark 2.4 (The shift alone). *[DERIVED] Without the homogeneous term the derivative terms of (4) vanish for $\alpha_1 = 0$, $\alpha_2 = 2\alpha_3$: the combination $I_\star = 2I_2 + I_3$ is invariant under the finite shift (3), and $\sqrt{-g}\phi^2 I_\star$ is exactly Weyl invariant without any gauge field. The three-parameter family $\sum_i \beta_i I_i[\tilde{D}]$ used below is therefore a choice that gives more freedom, not a necessity. The coefficients of A and B in (4) were checked by finite differences with the cyclic I_2 (`bvfrg_checks.py`).*

2.3 Weyl-invariant action, unitary gauge and the auxiliary sector

The ultraviolet action considered is

$$S_{\text{UV}} = \int d^4x\sqrt{-g}\left[\frac{1}{12}\phi^2\tilde{R} - \frac{1}{2}g^{\mu\nu}D_\mu\phi D_\nu\phi - \frac{1}{4g_V^2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4g_a^2}f_{\mu\nu}f^{\mu\nu} + \phi^2\sum_{i=1}^3\beta_i I_i[\tilde{D}] - \lambda\phi^4 + \bar{\psi}(i\gamma^\mu\hat{\nabla}_\mu - y\phi)\psi + \frac{3}{2}\tilde{a}_\sigma\bar{\psi}\gamma^\sigma\gamma_5\psi\right], \quad (5)$$

with $F = dV$, $f = d\tilde{a}$, and $\hat{\nabla}$ including the Levi-Civita spin connection. [ASSUMPTION] This action is a Weyl-invariant truncation, not a classification: the distortion enters only through $I_{1,2,3}$ (there are eleven independent parity-even quadratic invariants), the kinetic term f^2 of $\tilde{a}$ is added by hand, and the fermions couple through the Lorentz-covariant spin connection (so that only the axial part of D couples, with strength $3/2$; the vector part $P-R$ gives an anti-Hermitian term and $\hat{q}$ drops out). The β_i are free.

Auxiliary sector. In (5) the components of $\tilde{D}$ other than $\tilde{a}$ have no kinetic term. With $\tilde{P} = P - V$, $\tilde{Q} = Q - V$, $\tilde{R} = R + V$ and (2),

$$\sum_i \beta_i I_i[\tilde{D}] = -6(\beta_1 + \beta_2)\tilde{a}_\sigma \tilde{a}^\sigma + \sum_{ij} N_{ij} \tilde{x}_i \cdot \tilde{x}_j + (\hat{q} \text{ terms}),$$

$$N = \beta_1 \begin{pmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{pmatrix} + \beta_2 \begin{pmatrix} 1 & \frac{5}{2} & \frac{5}{2} \\ \frac{5}{2} & 1 & \frac{5}{2} \\ \frac{5}{2} & \frac{5}{2} & 1 \end{pmatrix} + \beta_3 uu^\top, \quad (6)$$

$\tilde{x} = (\tilde{P}, \tilde{Q}, \tilde{R})$, $u = (1, 1, 4)$. The form on $\hat{q}$ is $(\beta_1 + \beta_2)\hat{q}^2$ on the 16-dimensional totally symmetric traceless part and $(\beta_1 - \frac{1}{2}\beta_2)\hat{q}^2$ on the 32-dimensional complement. The eigenvalues of N for $\beta_3 = 0$ are $6(\beta_1 + \beta_2)$ and $3(\beta_1 - \frac{1}{2}\beta_2)$ (twice), and in general $\det N = \lambda_p^2 \lambda_s + 6\beta_3 \lambda_p (2\lambda_p + \lambda_s)$ with $\lambda_s = 6(\beta_1 + \beta_2)$, $\lambda_p = 3(\beta_1 - \frac{1}{2}\beta_2)$. The 64×64 quadratic form has no mixing between the axial, vector and traceless parts, and on the 60 non-axial components it is non-degenerate iff

$$\beta_1 + \beta_2 \neq 0, \quad 2\beta_1 \neq \beta_2, \quad \det N \neq 0 \quad (7)$$

(nullity 0 generically, 16 at $\beta_1 + \beta_2 = 0$ besides the 4 axial components whose mass then vanishes, 36 at $\beta_2 = 2\beta_1$; `bvfrg_checks.py`). Under (7) the field equations of these 60 components, which have no source in S_{UV} , give zero.

Unitary gauge. Setting $\phi = \phi_0$ the axial vector acquires $m_a^2 = 12(\beta_1 + \beta_2)g_a^2\phi_0^2$ (so that $\beta_1 + \beta_2 > 0$ is required) and the Weyl vector $m_V^2 = 2g_V^2\phi_0^2$, after rescaling to canonical normalisation; the Einstein–Hilbert term is $\frac{1}{12}\phi_0^2\hat{R}$, i.e. $M_{\text{Pl}}^2 = \phi_0^2/6$ with $G = 1/(8\pi M_{\text{Pl}}^2)$ and $\phi_0 = \sqrt{6} M_{\text{Pl}} \simeq 6 \times 10^{18}$ GeV for the reduced Planck mass. [DERIVED] The bosonic quadratic sector then has $2+3+3 = 8$ degrees of freedom (graviton, massive V_μ , massive $\tilde{a}_\mu$); this count is for the linearised bosonic theory around a background where the quadratic action is the one above. [OPEN] Non-linear (Boulware–Deser type) counting, curved backgrounds, fermions and quantum unitarity are not covered.

3 BRST–BV structure

3.1 Gauge algebra and BRST transformations

The gauge group is $\text{Diff} \ltimes \text{Weyl}$ (Weyl is a normal subgroup, $\omega \mapsto \xi^\mu \partial_\mu \omega$), times local Lorentz transformations when fermions are included; the latter commute with Weyl. With nested variations the infinitesimal transformations close, $[\delta_\xi, \delta_\eta] = -\delta_{[\xi, \eta]}$, $[\delta_\xi, \delta_\omega] = -\delta_{\xi^\mu \partial_\mu \omega}$, $[\delta_\omega, \delta_{\omega'}] = 0$, with field-independent structure constants, so the algebra closes off shell. The ghosts are c^μ , c_W and λ^{ab} (Lorentz). The BRST transformations are, with $\mathcal{L}_c$ the Lie derivative,

$$\begin{aligned} s g_{\mu\nu} &= \mathcal{L}_c g_{\mu\nu} + 2c_W g_{\mu\nu}, & s \Gamma_{\mu\nu}^\rho &= \mathcal{L}_c \Gamma_{\mu\nu}^\rho + \partial_\mu \partial_\nu c^\rho, \\ s \phi &= c^\nu \partial_\nu \phi - c_W \phi, & s V_\mu &= \mathcal{L}_c V_\mu - \partial_\mu c_W, \\ s c^\mu &= c^\nu \partial_\nu c^\mu, & s c_W &= c^\nu \partial_\nu c_W, \\ s e^a{}_\mu &= c^\nu \partial_\nu e^a{}_\mu + e^a{}_\nu \partial_\mu c^\nu + c_W e^a{}_\mu + \lambda^a{}_b e^b{}_\mu, & s \lambda^{ab} &= c^\nu \partial_\nu \lambda^{ab} + \lambda^{ac} \lambda_c{}^b, \\ s \psi &= c^\nu \partial_\nu \psi - \frac{3}{2} c_W \psi - \frac{1}{4} \lambda_{ab} \gamma^{ab} \psi, & s \bar{c}_\mu &= B_\mu, \quad s \bar{c}_W = B_W, \quad s B = 0, \end{aligned} \quad (8)$$

with $\{\gamma^a, \gamma^b\} = -2\eta^{ab}$, $\gamma^{ab} = \frac{1}{2}[\gamma^a, \gamma^b]$. Three points deserve emphasis: (i) $s\Gamma$ has the inhomogeneous term $\partial_\mu \partial_\nu c^\rho$ of the Lie derivative of a connection; its coefficient $+1$ is the only one for which $\nabla_\mu U^\rho$ transforms as a tensor; (ii) the spinor is a spacetime scalar, so it carries $c^\nu \partial_\nu \psi$, and the Lorentz term requires the vierbein; the relative sign between the Lorentz terms of e and ψ is not a convention, the opposite sign gives $s^2 \psi \neq 0$; (iii) $\tilde{D}$ and $\tilde{a}_\mu$ are composite ($D = \Gamma - \hat{\Gamma}[g]$, $\tilde{D} = D - W[V]$), so they carry no ghosts of their own. Ghosts of ghosts do not appear: the theory is irreducible and $\bar{c}$, B are antighosts and Nakanishi–Lautrup fields.

Proposition 3.1 (Nilpotency). *[DERIVED] $s^2 = 0$ on c^μ , c_W , λ^{ab} , $g_{\mu\nu}$, e^a_μ , $\Gamma^\rho_{\mu\nu}$, ϕ , V_μ and ψ .*

Proof. Let $\delta_A \Phi$ be variations with $[\delta_A, \delta_B] = f^C_{AB} \delta_C$ (nested variation), field-independent f , and set $s\Phi = c^A \delta_A \Phi$, $sc^C = -\frac{1}{2} f^C_{AB} c^A c^B$. Since c is odd and s acts on the fields inside $\delta_A \Phi$ as $c^B \delta_B$, $s^2 \Phi = (sc^A) \delta_A \Phi - c^A c^B \delta_B \delta_A \Phi = (sc^C + \frac{1}{2} f^C_{AB} c^A c^B) \delta_C \Phi = 0$; on the ghosts $s^2 c^C$ is proportional to the Jacobi identity of f . For the diffeomorphism ghost, $s^2 c^\mu = -c^\nu c^\rho \partial_\nu \partial_\rho c^\mu = 0$ because $c^\nu c^\rho$ is antisymmetric (for commuting c it would not vanish). For the cases in which the inhomogeneous term of Γ or the spinor enters, nilpotency is not visible by inspection and was verified in an exact graded algebra (Grassmann-odd ghost jets, rational coefficients, $d = 2, 3, 4$, jet order up to 5; for $d = 4$ Lorentzian signature with an 8×8 real Clifford representation); the negative controls fail as they must (commuting ghosts, $sc^\mu = \pm 2c \cdot \partial c$, $sc_W = 0$, the wrong sign of $\partial \partial c$ in $s\Gamma$, the wrong relative sign of the Lorentz term in $s\psi$): `partIII_brst.py`. $\square$ $\square$

3.2 Antifields, classical master equation and gauge fixing

To each field and ghost Φ^A one assigns an antifield Φ_A^* [STANDARD] [13] with $\epsilon(\Phi^*) = \epsilon(\Phi) + 1$, $\text{gh}(\Phi^*) = -\text{gh}(\Phi) - 1$ and Weyl weight $w(\Phi^*) = -w(\Phi)$ (antifields are densities, so that $\int \Phi^* s\Phi$ needs no metric); every term $\Phi_A^* s\Phi^A$ then has Grassmann parity, ghost number and weight zero (checked row by row in `partIII_brst.py`). The BV action is

$$S_{BV} = S_{\text{cl}}[\Phi] + \int d^4x \left(\Phi_A^* s\Phi^A + c_\mu^* sc^\mu + c_W^* sc_W + \lambda_{ab}^* s\lambda^{ab} + \bar{c}^{*\mu} B_\mu + \bar{c}_W^* B_W \right), \quad S_{\text{cl}} = S_{\text{UV}}, \quad (9)$$

and $s\Phi^A = (\Phi^A, S_{BV})$ for the BV bracket $(F, G) = F_{,A} G^{,A} - F^{,A} G_{,A}$ (up to Grassmann signs).

Theorem 3.2 (Classical master equation). *[DERIVED] $(S_{BV}, S_{BV}) = 0$.*

Proof. Write $S_{BV} = S_{\text{cl}} + \Phi_A^* R^A[\Phi]$ with $R^A = s\Phi^A$, which does not depend on the antifields. Then $(S_{BV}, S_{BV}) = 2S_{BV,A} R^A$ has two terms. At order zero in the antifields it is $S_{\text{cl},A} R^A$, the variation of S_{cl} under the gauge transformation with parameters replaced by ghosts, which vanishes because S_{cl} is invariant under Diff, Weyl and Lorentz. At order one it is $\Phi_B^* R^B{}_{,A} R^A = \Phi_B^* s(\Phi^B) = 0$ by Proposition 3.1. There are no higher orders because R^A is independent of the antifields: this, and not the field independence of the structure constants (the transformations of g, Γ, V are field dependent), is why S_{BV} is linear in the antifields. $\square$ $\square$

Gauge fixing. Use the background split $g = \bar{g} + h$ and the fermion $\Psi = \int \sqrt{\bar{g}} [\bar{c}^\nu (F_\nu + \frac{\xi}{2} B_\nu) + \bar{c}_W (\phi - \phi_0)]$, $F_\nu = \bar{\nabla}^\mu h_{\mu\nu} - \frac{1}{2} \bar{\nabla}_\nu h$ (the harmonic gauge of the flow in Section 6). Eliminating $\Phi^* = \delta\Psi/\delta\Phi$ gives $S_{\text{FP}} = S_{\text{cl}} + s\Psi$; B_W enforces $\phi = \phi_0$ (unitary gauge). Because $s\phi|_{\phi_0} = -\phi_0 c_W$ does not involve c^μ for constant ϕ_0 , the ghost operator in the basis $(\bar{c}^\nu, \bar{c}_W) \times (c^\mu, c_W)$ is triangular,

$$\begin{pmatrix} \Delta_{\text{FP}}{}^\nu{}_\mu & 2\partial^\nu \\ 0 & \phi_0 \end{pmatrix}, \quad \det = \det \Delta_{\text{FP}} \cdot \det \phi_0, \quad (10)$$

and the second factor is ultralocal and field independent. [DERIVED] In unitary gauge the Weyl ghosts decouple and only the diffeomorphism ghosts propagate. In the symmetric phase an R_ξ -type gauge is needed to cancel the V - χ and χ - h mixings; it is not constructed here. [OPEN]

4 BV functional renormalization group

4.1 Regulator and its BRST variation

Euclidean conventions, $\hbar$ kept. Collect all fields, ghosts, antighosts and NL fields in Φ^A and write $R^A = s\Phi^A$ (independent of antifields). Let $S_\Psi = S_{\text{cl}} + s\Psi$ be the gauge-fixed action, so that $R^A \partial_A S_\Psi = 0$ and $R^A \partial_A R^B = 0$ (nilpotency). A cutoff term is $\Delta S_k = \frac{1}{2} \Phi^A R_{k,AB} \Phi^B$, with R_k built from the background fields. Its BRST variation is $s\Delta S_k = R_{k,AB} \Phi^B R^A$. It does *not* vanish in general: for diffeomorphisms R^A is inhomogeneous or non-linear in the fields (for h , $sh = \mathcal{L}_c(\bar{g} + h)$), and for Weyl $R^A \supset w_A c_W \Phi^A$ while R_k is built from a Weyl-non-covariant Laplacian. A regulator with $s\Delta S_k = 0$ cannot be assumed; the symmetry breaking is instead kept in the identity below.

4.2 The modified Slavnov–Taylor identity

Modified Slavnov–Taylor identities for flows with a symmetry-breaking regulator are known [STANDARD] [14, 15, 16]; the derivation below is the standard argument, written for a general BRST vector field R^A that is independent of the antifields.

Introduce sources J_A for the fields and Φ_A^* for the BRST variations:

$$Z_k[J, \Phi^*] = \int D\Phi \exp \left\{ -\frac{1}{\hbar} [S_\Psi + \Phi_A^* R^A + \Delta S_k - J_A \Phi^A] \right\}, \quad W_k = \hbar \ln Z_k, \quad (11)$$

$\varphi^A = \delta W_k / \delta J_A$, $\Gamma_k[\varphi, \Phi^*] = -W_k + J_A \varphi^A - \Delta S_k[\varphi]$, so that $J_A = \Gamma_{k,A} + R_{k,AB} \varphi^B$, $\delta \Gamma_k / \delta \Phi_A^* = \langle R^A \rangle$, $\Gamma_k = S_{BV}$ at tree level for $k \rightarrow \infty$ and

$$\partial_t \Gamma_k = \frac{\hbar}{2} \text{STr} \left[\partial_t R_k (\Gamma_k^{(2)} + R_k)^{-1} \right], \quad t = \ln k, \quad (12)$$

the Wetterich equation ([STANDARD] [5]; the antifields are spectators because R_k does not depend on them).

Proposition 4.1 (Modified Slavnov–Taylor identity). *[DERIVED] Let the BRST vector field R^A be independent of the antifields and nilpotent. Then, with $(\Gamma_k, \Gamma_k) = 2 \Gamma_{k,A} \Gamma_k^{*,A}$,*

$$\frac{1}{2}(\Gamma_k, \Gamma_k) = \hbar \mathcal{K}_k - \hbar \mathcal{J}, \quad \mathcal{K}_k = R_{k,AB} (\Gamma_k^{(2)} + R_k)^{-1 BC} \frac{\delta^2 \Gamma_k}{\delta \Phi_A^* \delta \varphi^C}, \quad \mathcal{J} = \langle \partial_A R^A \rangle. \quad (13)$$

Proof. The integral of a total derivative vanishes: $0 = \int D\Phi \partial_A (R^A e^{-\Sigma/\hbar})$, $\Sigma = S_\Psi + \Phi^* R + \Delta S_k - J\Phi$. Now $R^A \partial_A S_\Psi = 0$ and $R^A \partial_A (\Phi_B^* R^B) = \Phi_B^* s^2 \Phi^B = 0$, so

$$0 = \hbar \langle \partial_A R^A \rangle - \langle R^A R_{k,AB} \Phi^B \rangle + J_A \langle R^A \rangle. \quad (14)$$

Insert $J_A = \Gamma_{k,A} + R_{k,AB} \varphi^B$ and $\langle R^A \rangle = \Gamma_k^{*,A}$: $\Gamma_{k,A} \Gamma_k^{*,A} = -\hbar \langle \partial_A R^A \rangle + R_{k,AB} \langle R^A (\Phi^B - \varphi^B) \rangle$. The connected correlator is $\langle R^A (\Phi^B - \varphi^B) \rangle = \hbar \delta \langle R^A \rangle / \delta J_B = \hbar \Gamma_{k,*,A}^C (\Gamma_k^{(2)} + R_k)^{-1 CB}$, since $\delta \varphi^C / \delta J_B = (\Gamma_k^{(2)} + R_k)^{-1 CB}$. This is (13). $\square$

Properties. (i) For $k \rightarrow \infty$, $R_k(\Gamma_k^{(2)} + R_k)^{-1} \rightarrow 1$ and $\Gamma_k \rightarrow S_{BV}$, so $\mathcal{K}_k \rightarrow \mathcal{J}$ and both sides vanish, consistently with Theorem 3.2. (ii) $\mathcal{K}_k$ is the regulator-induced breaking: it is a one-loop-type expression in the full propagator, exact (non-perturbative) and explicit; it is not a quantity “defined as whatever restores the identity”. (iii) $\mathcal{J}$ does not involve the regulator. Since R^A is affine in the fields, $\partial_A R^A$ depends at most on the ghosts, with a coefficient $\propto \delta^4(0)$. For diffeomorphisms and Lorentz transformations it vanishes in a regularisation that respects them; for Weyl transformations $\mathcal{J} \supset \delta^4(0) \int c_W \sum_A (-1)^{\epsilon_A} w_A$, and a covariant regularisation

leaves a finite local remainder $\int c_W \mathcal{A}$ with $\mathcal{A}$ of dimension four. This remainder is the object whose coefficients are the anomaly of the gauged symmetry (Section 5); in this formulation *the anomaly does not flow, only $\mathcal{K}_k$ does*. (iv) In unitary gauge the Weyl ghosts decouple (10), so no regulator is needed for them.

4.3 The third equation

Proposition 4.2 (Integrability, “third equation”). *[DERIVED] If (13) holds for all k , then*

$$\boxed{\hbar \partial_t \mathcal{K}_k = (\Gamma_k, \partial_t \Gamma_k) = \frac{\hbar}{2} \left(\Gamma_k, \text{STr} \left[\frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k} \right] \right)}. \quad (15)$$

Proof. Differentiate (13) with $\partial_t \mathcal{J} = 0$ (item (iii)), use $\partial_t(\Gamma_k, \Gamma_k) = 2(\Gamma_k, \partial_t \Gamma_k)$ and (12). $\square \quad \square$

Remark 4.3 (Status of the third equation). *[DERIVED]* Both (12) and (13) follow from the same regulated path integral, so the flow started at $\Gamma_\Lambda = S_{BV}$ preserves the mSTI: (15) is then an identity and carries no information beyond the other two. Its use is as a diagnostic. For a truncated ansatz Γ_k^{tr} the functional $\Sigma_k[\Gamma] = \frac{1}{2}(\Gamma, \Gamma) - \hbar \mathcal{K}_k + \hbar \mathcal{J}$ does not vanish, and its size measures the violation of the symmetry by the truncation. The Einstein–Hilbert flow of Section 6 uses type-A cutoffs for gravity, evaluates the trace at $\hbar = 0$ (a single metric) and has no antifield dependence; it solves (12) projected on two couplings and does not impose (13). Its violation, $\Sigma_k \neq 0$, has not been computed. *[OPEN]*

The explicit trace entering (12) for the Einstein–Hilbert projection with matter is given in Section 6.

5 Anomaly structure

Three different objects are called “anomaly” and must be kept apart. (A) The trace anomaly of a conformal field theory in a background metric, with coefficients (c, a, b) in $\langle T^\mu{}_\mu \rangle = \frac{1}{16\pi^2}(c C^2 - a E_4 + b \square R)$. (B) The logarithmic one-loop divergence $\text{tr } a_2$, defined for any Laplace-type operator; for non-conformal operators it contains R^2 and renormalises couplings. (C) The anomaly of the gauged Weyl symmetry of QDG, an obstruction to the quantum master equation, a cocycle $\int c_W \mathcal{A}$ of ghost number one. The coefficients of C^2 and E_4 in (B) do not depend on masses or on non-derivative couplings; (C) is not known.

5.1 Coefficients of the matter content

In the basis $\text{tr } a_2 = c C^2 - a E_4 + \varrho R^2 + b \square R$ one finds (exact fractions, `partIII_anomaly.py`, `bvfrg_checks.py`):

operator	c	a	ϱ
conformal real scalar	1/120	1/360	0
Dirac fermion	6/120	11/360	0
Maxwell field	12/120	62/360	0
Proca field, $m \rightarrow 0$	13/120	63/360	1/72
2 Proca + n_f Dirac	$(26 + 6n_f)/120$	$(126 + 11n_f)/360$	1/36
$n_f = 3$	11/30	53/120	1/36

The first three rows are standard *[STANDARD]* [12]. The Proca row follows from the exact determinant of a massive vector on a fixed background, $\det(\Delta_1 + m^2)^{-1/2} \det(\Delta_0 + m^2)^{1/2}$, i.e. a Maxwell field plus one *minimally* coupled Stueckelberg scalar: (c, a) equal those of Maxwell

plus a scalar, but the scalar is not conformal and leaves $\varrho = 1/72$ [DERIVED]. The massless limit is the regime curvature $\gg$ mass; c, a do not depend on m .

Remark 5.1 (Coefficients that are not valid). [DERIVED] Assigning the Maxwell coefficients (12/120, 62/360) to the two Proca fields, or a graviton entry $c = 0, a = 212/360$, is not correct; such assignments would give, for $n_f = 3, c = 7/20$ and $a = 41/40$. The graviton has no pair (c, a) : Einstein gravity is not Weyl invariant, and $c_{\text{grav}} = 0$ “because the classical action has no C^2 term” does not follow, since c is fixed by the field content of the loop. The number 212 is the numerator of the on-shell quantity $4 \cdot \frac{53}{45} = \frac{212}{45}$, obtained for the graviton-plus-ghost determinant on a Ricci-flat background in de Donder gauge (`partIII_anomaly.py`); it is a Euler-type coefficient valid on Ricci-flat backgrounds only, and off shell the coefficients of that determinant depend on the gauge and on the parametrisation of the metric. The totals for the full UV content (graviton, dilaton, Weyl vector, axial vector, fermions) are not computed. [OPEN]

5.2 The auxiliary sector is ultralocal

Proposition 5.2 (Ultralocal block). [DERIVED] Assume (7). In S_{UV} the 60 components $(\tilde{P}, \tilde{Q}, \tilde{R}, \hat{q})$ of $\tilde{D}$ appear only in $\phi^2 \sum_i \beta_i I_i[\tilde{D}]$, with a quadratic form that has no mixing with $\tilde{a}_\mu$, and no mixing with any other field. Their Gaussian integral is ultralocal, with $\mathcal{N}$ the 60×60 matrix of the form (6) including the $\hat{q}$ blocks,

$$\int D\tilde{x} e^{-\int \phi^2 \tilde{x}^T \mathcal{N} \tilde{x}} \propto \exp \left\{ -\frac{1}{2} \delta^{(4)}(0) \int d^4x \ln \det(\phi^2 \mathcal{N}) \right\}, \quad (16)$$

(Euclidean continuation), with a local scalar density of dimension four and no curvature dependence. Hence it contributes to none of c, a, ϱ, b in $\text{tr } a_2$.

Proof. $\tilde{D} = D - W[V]$ is a V -dependent shift of the integration variable, with unit Jacobian. The Dirac coupling involves only $\tilde{a}$, and $\tilde{R}$ involves V only; the quadratic form has no mixing between the axial, vector and traceless parts (`bvfrg_checks.py`), and no derivative acts on $\tilde{P}, \tilde{Q}, \tilde{R}, \hat{q}$ (their propagator is a constant, a contact term). An operator without derivatives is not of Laplace type; its determinant is $\prod_x \det(\phi^2 \mathcal{N})$, proportional to $\delta^{(4)}(0) \int d^4x (\dots)$, which is a vacuum-energy-type term. In dimensional or zeta-function regularisation $\delta^{(4)}(0) = 0$; with a cutoff, $\delta^{(4)}(0) \sim \Lambda^4/16\pi^2$ is curvature independent, and if the cutoff is tied to the dilaton, $\Lambda \propto \phi$, the term is $\propto \sqrt{-g}\phi^4$, i.e. it renormalises λ and preserves Weyl invariance. $\square$ $\square$

This is the correct form of the statement that “the distortion sector does not contribute to the anomaly”: it holds for the 60 auxiliary components, not for $\tilde{a}_\mu$. The axial vector is a Proca field with kinetic term f^2 and mass m_a ; it is counted in the table above (as part of the two Proca fields). [ASSUMPTION] That the Weyl vector V_μ and the axial vector are both treated as Proca fields is the counting of the infrared theory (Section 6). The proposition says nothing about object (C): the Jacobian $\mathcal{J}$ of the auxiliary components is the same $\delta^{(4)}(0)$ type, but the full cocycle of $\text{Diff} \ltimes \text{Weyl}$ with dynamical V_μ, ϕ and Γ is open. [OPEN]

5.3 What changes if curvature-squared terms of the connection are added

Curvature-squared terms of the independent connection, $S_R = \int \sqrt{-g} [\alpha_1 R(\Gamma)^2 + \alpha_2 R_{\mu\nu}(\Gamma) R^{\mu\nu}(\Gamma) + \alpha_3 R_{\mu\nu\rho\sigma}(\Gamma) R^{\mu\nu\rho\sigma}(\Gamma)]$, are *not* part of S_{UV} . They are considered here as an extension. Linearising in D , $\delta R^\rho_{\sigma\mu\nu}|_D = \hat{\nabla}_\mu D^\rho_{\nu\sigma} - \hat{\nabla}_\nu D^\rho_{\mu\sigma}$, hence $\delta R|_D = \hat{\nabla} \cdot C - \hat{\nabla} \cdot A$, and with $A = 4P + Q + R$, $C = P + Q + 4R$,

$$\delta R|_D = 3 \hat{\nabla}_\rho (R^\rho - P^\rho), \quad (17)$$

a total derivative [DERIVED] (checked algebraically, `bvfrg_checks.py`). More important is the quadratic term: $R^\rho_{\sigma\mu\nu}(\Gamma) = \hat{R}^\rho_{\sigma\mu\nu} + (\hat{\nabla}_\mu D^\rho_{\nu\sigma} - \hat{\nabla}_\nu D^\rho_{\mu\sigma}) + DD$, so for $\alpha_3 \neq 0$ the action

contains $\alpha_3(\hat{\nabla}_\mu D^\rho{}_{\nu\sigma} - \hat{\nabla}_\nu D^\rho{}_{\mu\sigma})^2$, a two-derivative kinetic term for D . Its principal symbol $D \mapsto k_\mu D^\rho{}_{\nu\sigma} - k_\nu D^\rho{}_{\mu\sigma}$ has rank 48 on the 64 components (kernel of dimension 16). [DERIVED]

Remark 5.3 (Consequences). [DERIVED] (the last two sentences are power-counting estimates) For $\alpha_i \neq 0$ most of the auxiliary components become dynamical, Proposition 5.2 no longer applies, the degree-of-freedom count of Section 2 (eight) is no longer valid, and mixing terms $h \hat{R} \hat{\nabla} D$ must be treated together with the new propagator $1/(\alpha_3 k^2 + \phi_0^2 \beta)$, not with M^{-1} alone. Moreover the anomaly coefficients c, a are numbers fixed by the field content; a correction proportional to a power of the background curvature $\bar{R}$ (for instance of order $\alpha_3^2 \bar{R}^2 / \phi_0^4$ from a mixing of strength $\alpha_3 \bar{R}$) is a higher-dimension operator, suppressed by powers of ϕ_0^{-2} , not a shift of a . A relation $\delta a_E / a_E^{\text{geom}} = \alpha_3^4 f$ with f a pure number, and the vanishing of an effective endomorphism on de Sitter space, are therefore not supported by this analysis. A consistent treatment of the extension, including the new degrees of freedom and ghosts, is an open problem. [OPEN]

5.4 Where anomaly coefficients enter the flow

The Einstein–Hilbert projection of the Wetterich equation reads off the coefficient of the volume term and of R in $\partial_t \Gamma_k$, which involve a_0 and a_1 of the heat-kernel expansion; the invariants C^2 and E_4 appear at the next order, in $\text{tr } a_2$. The Weyl-anomaly coefficients (c, a) therefore do not enter the flow of g and λ . The coefficient that does enter a truncation with R^2 is ϱ (object (B)), not a : identifying the coefficient of the Euler density with that of R^2 in $\beta_{g_{R^2}}$, as in $\beta_{g_{R^2}} \ni a_E / (4\pi)^2$, is not correct. Computing the matter contribution to the R^2 coupling in the Lauscher–Reuter truncation [9] for QDG-like content is [OPEN].

6 The flow and the fixed point

6.1 The derived Einstein–Hilbert flow with matter

The flow used here is the Einstein–Hilbert flow of the Wetterich equation on S^4 , Litim cutoff, harmonic gauge $\alpha = 1$ (the gauge function F_ν of Section 3) and linear split, with $g = k^2 G_k$, $\lambda = \bar{\Lambda}_k / k^2$, $u = 1 - 2\lambda$, $\eta = \partial_t \ln G_k$. Three inputs are taken from the literature and not rederived [STANDARD]: the Hessian constants of the Einstein–Hilbert action in this gauge [6], the heat-kernel coefficient $\frac{1}{6}R$ for Bochner Laplacians [11], and the threshold-function formalism [7, 8]. The remaining steps are computed here [DERIVED]:

$$\begin{aligned} \beta_g &= (2 + \eta)g, & \eta &= \frac{g[B_1(\lambda) + X/(6\pi)]}{1 - g B_2(\lambda)}, \\ \beta_\lambda &= -(2 - \eta)\lambda + \frac{g}{2\pi} \left[\frac{5}{u} - 4 - \frac{5\eta}{6u} + \frac{Y}{2} \right], \\ B_1 &= \frac{1}{3\pi} \left[\frac{5}{u} - \frac{9}{u^2} - 7 \right], & B_2 &= -\frac{1}{12\pi} \left[\frac{5}{u} - \frac{6}{u^2} \right], \end{aligned} \tag{18}$$

where all ten metric modes have the common mass -2λ , the ghost operator is $-\bar{D}^2 - \frac{1}{4}R$, and the matter enters through two integers,

$$X = N_S + 2N_D - 4N_V - 3N_P, \quad Y = N_S - 4N_D + 2N_V + 3N_P \tag{19}$$

(N_S real scalars, N_D Dirac fermions, N_V massless vectors, N_P Proca fields in the limit $m \rightarrow 0$, a massless vector plus one scalar; matter is minimally coupled, with a cutoff that is a function of the full fluctuation operator, *type B*, for which the values of X depend on this choice). Closed forms of $B_{1,2}$ agree with the numerical threshold integrals to 10^{-10} (`eh_flow_trace.py`). Pure gravity has $(g_*, \lambda_*) = (0.7073, 0.1932)$, $\theta = 1.475 \pm 3.043 i$, the standard Einstein–Hilbert result for this cutoff [10].

Proposition 6.1 (Closed form of the fixed point). *[DERIVED] The non-Gaussian fixed points of (18) are $\lambda_* = \frac{1}{2}(1 - u)$, $g_* = 2\pi/N(u)$, $N(u) = \frac{4}{u^2} - \frac{5}{2u} + \frac{c}{6}$, $c = 14 - X$, where u is a real root of*

$$2cu^3 - (2c + 54 - 3Y)u^2 + 118u - 48 = 0 \quad (20)$$

with $u > 0$, $N(u) > 0$ and $\frac{3}{u^2} - \frac{5}{3u} + \frac{c}{6} > 0$ (i.e. $1 - g_*B_2 > 0$).

Proof. $\beta_g = 0$ with $g \neq 0$ gives $\eta = -2$; inserting in the expression for η gives $g_* = 2\pi/N(u)$. With $\eta = -2$ the second equation reads $4\lambda = \frac{g}{2\pi}[\frac{20}{3u} - 4 + \frac{Y}{2}]$; inserting g_* , multiplying by $6u^2N(u)$ and using $\lambda = (1 - u)/2$ gives $2(1 - u)(24 - 15u + cu^2) = u(40 + (3Y - 24)u)$, which is (20). $\square$

6.2 Matter content of QDG and the counting

[ASSUMPTION] In the infrared theory the dynamical fields beyond the graviton are two massive vectors (the Weyl vector V_μ and the axial vector $\tilde{a}_\mu$) and n_f Dirac fermions. In the limit $m^2 \ll k^2$ each Proca field is a massless vector plus one scalar, so that

$$X = 2n_f - 6, \quad Y = 6 - 4n_f, \quad \text{i.e.} \quad (40 - 4n_f)u^3 - (76 + 8n_f)u^2 + 118u - 48 = 0. \quad (21)$$

This counting is consistent with the ultraviolet field content: the Weyl vector together with the dilaton is a massless vector plus a scalar, which is what the massive V_μ is after unitary gauge. Treating the two vectors as massless instead gives $X = 2n_f - 8$, $Y = 4 - 4n_f$; the two coefficients must be computed with the same treatment of the vectors. The following is not in this flow: the operators of the Weyl-gauged sector (non-minimal couplings of V_μ , ϕ and $\tilde{a}_\mu$ to curvature, the mixing of the dilaton with the metric in the symmetric phase, the auxiliary sector) do not enter. A scalar with coupling $\xi R\varphi^2$ would have $X = 1 - 6\xi$ and $Y = 1$ in the same scheme, so non-minimal couplings shift X directly. [OPEN] Whether and how the QDG-specific operators modify η and β_λ is an open problem.

6.3 Results for the QDG-like content

For $n_f = 3$ the real root of (21) is $u = 1.6358$, and the full flow with threshold functions by quadrature gives $\eta = -2.0000000000$, $\beta_\lambda = 4 \times 10^{-16}$ there; the other two roots are complex. [DERIVED] (Litim, $a = 1$, two Proca fields, massless limit; `bvfrg_fixed_point.py`, `eh_flow_trace.py`):

n_f	X	Y	g_*	λ_*	critical exponents θ_i
0	-6	6	0.5093	0.2290	$2.171 \pm 3.263 i$
1	-4	2	0.6204	0.2030	$1.760 \pm 3.023 i$
2	-2	-2	0.8362	0.1568	$1.226 \pm 2.531 i$
3	0	-6	2.7319	-0.3179	0.723, 2.207
4	2	-10	3.9037	-1.0676	1.797, 3.161
6	6	-18	5.9848	-2.8594	2.146, 3.695
8	10	-26	11.836	-7.8175	2.249, 3.899
9.9	13.8	-33.6	233.5	-193.1	2.293, 3.996

The branch is continuous in n_f ; λ_* passes through zero at $n_f = 17/6$ and the exponents turn from a complex pair to two real positive numbers at $n_f \simeq 2.87$. For $n_f \geq 10$ no admissible root exists: at $X = 14$ ($c = 0$) one root escapes to infinity ($u \simeq (54 - 3Y)/2c$, $g_* \sim \lambda_* \sim 1/c$), and the real root left for $n_f > 10$ has $N(u) < 0$; this is a property of these one-parameter families (a scan from $n_f = 10$ to 200), and the boundary is exactly $X = 14$ only for $Y < -15.36$, since for larger Y another root survives beyond $X = 14$ [DERIVED]. Near the end of the branch $\lambda_* \simeq -19.5/(10 - n_f)$ and $g_* \simeq (52\pi/7)/(10 - n_f)$. The fixed point at $n_f = 3$, $g_* = 2.73$,

$\lambda_* = -0.32$, is outside the range $g_* < 2$, $\lambda_* > -1$ in which the truncation is usually regarded as reliable (a convention, not a derived bound); the critical number $n_{\text{crit}} = 10$ is the point where the fixed point runs off to infinity, an indicator of where the truncation stops being informative, not a physical limit on the number of species.

6.4 Regulator-robust existence

A necessary condition for a fixed point to be a property of the theory rather than of the regulator is that it exist for all members of a family of regulators (regulator-robust existence, RRE): here an *admissible* fixed point has $g_* > 0$, $1 - g_* B_2 > 0$ and $\text{Re } \theta_i > 0$, and ρ is the fraction of the nine regulators (Litim $R^{(0)} = a(1 - z)\theta(1 - z)$ with $a = 0.5, 0.75, 1, 1.5, 2, 3$ and exponential $R^{(0)} = az/(e^z - 1)$ with $a = 0.5, 1, 2$) for which one exists. RRE does not address gauge or parametrisation dependence, nor whether the fixed point lies where the truncation can be trusted. Along the QDG-like line [DERIVED] (`bvfrg_rre_qdg.py`, using the regulator scripts of the monograph [2]):

n_f	0	1	2	3	4	6	8	9	9.5	10	10.5
ρ	1	1	1	1	1	1	1	1	0.78	0.22	0
$\max g_*$	0.61	0.74	0.98	2.83	4.9	7.5	13.9	34.5	66.8	94.1	–
$\min \lambda_*$	0.17	0.15	0.12	−0.64	−3.0	−8.2	−21	−51	−69	−49	–

(maximum and minimum over the nine regulators). Existence of the fixed point with the minimal-coupling counting is regulator-robust up to $n_f \simeq 9$; for $n_f \leq 2$ it lies in the conventional range for all regulators; for $n_f = 3$ it is already just outside, and it moves away quickly with n_f . For two massless vectors the corresponding numbers are $\rho = 1$ up to $n_f = 10$ and 0.22 at 11. In the R^2 truncation of Lauscher and Reuter the non-Gaussian fixed point of pure gravity generalises to the enlarged theory space [9]; a scan of regulators in that truncation with Standard-Model matter exists in the QDG monograph [2] (those numbers were not recomputed for this paper). It has not been done for QDG-like content. [OPEN]

6.5 Dependence on the masses of the Proca fields

[ASSUMPTION] Let the dimensionless masses $w = m^2/k^2$ of the two Proca fields (and w_ψ for the fermions) be parameters. In the Litim scheme with matter regulated as in type B, with the mass added to the regulated operator, every threshold function is $\Phi_n^1(w) = \Phi_n^1(0)/(1 + w)$, so that

$$X = \frac{2n_f}{1 + w_\psi} - \frac{3}{1 + w_V} - \frac{3}{1 + w_a}, \quad Y = -\frac{4n_f}{1 + w_\psi} + \frac{3}{1 + w_V} + \frac{3}{1 + w_a}. \quad (22)$$

For $w_\psi = 0$ and a common w the boundary $X = 14$ (valid since $Y \leq -28$ there) gives $n_{\text{crit}}(w) = 7 + 3/(1 + w)$, confirmed by bisection on the existence of a root for $w = 0, 0.5, 1, 3, 10$ (`bvfrg_fixed_point.py`). [DERIVED] At $n_f = 3$:

w	X	Y	g_*	λ_*	θ_i
0	0	−6	2.732	−0.318	0.723, 2.207
0.5	2	−8	3.873	−0.853	1.619, 2.923
1	3	−9	4.355	−1.115	1.807, 3.113
5	5	−11	5.500	−1.756	2.039, 3.390
∞	6	−12	6.246	−2.176	2.116, 3.497

Making the Proca fields heavy removes them from the flow and the fixed point moves further out; the decoupling limit is three Dirac fermions with gravity. The masses are not free of the flow: if the Planck mass at scale k is identified with the dilaton value, $G_k = 6/(8\pi\phi_0^2)$ [ASSUMPTION],

then $\phi_0^2/k^2 = 3/(4\pi g)$ and $w_V = 3g_V^2/(2\pi g_*) = 0.175 g_V^2$, $w_a = 9(\beta_1 + \beta_2)g_a^2/(\pi g_*) = 1.05(\beta_1 + \beta_2)g_a^2$ at $n_f = 3$; $w_V = 1$ would require $g_V^2 \simeq 5.7$. The couplings g_V , g_a , β_i are free parameters of QDG, so w is not fixed.

6.6 Sensitivity to unknown contributions

The Weyl-gauged sector is not in the flow. As a bookkeeping device [ASSUMPTION] its unknown contribution can be represented by effective shifts $(\delta X, \delta Y)$, which captures contributions with the functional form of minimally coupled matter but not those that modify $B_{1,2}$. The existence region of the cubic in the (X, Y) plane is known exactly for Litim $a = 1$: for $Y < -15.36$ it is bounded by $X < 14$; for larger Y it extends to larger but bounded X (for example up to $X = 277$ at $Y = -6$ and 340 at $Y = 0$). Starting from the $n_f = 3$ point $(0, -6)$ and keeping $Y = -6$, a fixed point exists for all $X \in (-400, 277]$ and, with $\text{Re } \theta_i > 0$ required, for $X \leq 13.75$; at $X = 0$ it exists for all Y scanned ($|Y| \leq 200$). Added minimally coupled scalars, massless vectors or Proca fields do not remove it up to 400 extra fields; extra Dirac fermions do, at 7 additional ones ($n_f = 10$) (`bvfrg_tolerance.py`, `bvfrg_fixed_point.py`). [DERIVED] These statements concern *existence* in the minimally coupled Einstein–Hilbert flow; they say nothing about the reliability of the truncation at the resulting couplings.

6.7 Relation to the phenomenological flow

Earlier QDG analyses [3, 4] used a one-loop system with $\eta = -\tilde{G}A(\lambda)/(1 - \tilde{G}B(\lambda)) + C_\eta \tilde{G}$, $\beta_\lambda = -(2 - \eta)\lambda + \tilde{G}B(\lambda)/2\pi + C_{\text{bl}}\tilde{G} + 72\lambda^2/16\pi^2$, with $A = 5/(1 - 2\lambda) - 3/(1 - \lambda) - 1$, $B = A + 1$, $C_\eta = (n_f - 4)/3\pi$, $C_{\text{bl}} = (3 - 2n_f)/2\pi$. [MODEL] These functions have the form of signed multiplicities of the *volume* term and are used as the coefficient of R ; the $72\lambda^2$ term is not derived from the action; C_η corresponds to massless vectors and C_{bl} to Proca fields. Its numbers are internally consistent and reproduced by `bvfrg_model_flow_check.py`, but it is not the Einstein–Hilbert flow, and in the pure-gravity limit it does not reproduce its fixed point:

	model flow [MODEL]	derived flow [DERIVED]
pure gravity (g_*, λ_*)	(0.350, 0.031)	(0.707, 0.193)
$n_f = 3$ (g_*, λ_*)	(0.432, -0.019)	(2.732, -0.318)
$n_f = 3$ exponents	3.519, 9.446	0.723, 2.207

No rescaling of g or λ maps one onto the other (the model has an extra pole at $\lambda = 1$). Consequently the value $G_* = 0.43$, $\lambda_* = -0.02$ is not a derived property of QDG and is not used.

7 Discussion

Statement	Status
BRST algebra of $\text{Diff} \ltimes \text{Weyl} (\times \text{Lorentz})$, $s^2 = 0$ (Prop. 3.1)	[DERIVED], exact script
Classical master equation (Thm. 3.2); unitary-gauge ghost determinant (10)	[DERIVED]
Quantum master equation for QDG; anomaly of the gauged Weyl symmetry	[OPEN]
mSTI with explicit regulator term (Prop. 4.1); flow preserves it	[DERIVED]
Third equation (Prop. 4.2)	[DERIVED], integrability, no new content
Violation of the mSTI by the truncated flow of Section 6	not computed [OPEN]
60 auxiliary components ultralocal (Prop. 5.2)	[DERIVED] for S_{UV} ; fails for $\alpha_i \neq 0$
(c, a, ϱ) of Proca and Dirac matter; value for $n_f = 3$	[DERIVED]
(c, a) of the graviton; totals for the full UV content	undefined / [OPEN]
Einstein–Hilbert flow with matter, cubic (20)	[DERIVED] (Hessian input [STANDARD])
Fixed point of the QDG-like content; $n_{\text{crit}} = 10$; RRE up to $n_f \simeq 9$	[DERIVED], minimal-coupling counting [ASSUMPTION]
Fixed point at $n_f = 3$ within the range where the truncation is reliable	no ($g_* = 2.73$)
Effect of the Weyl-gauged operators on the flow	[OPEN]
Gauge ($\alpha \neq 1$) and parametrisation dependence	not tested [OPEN]
Phenomenological flow of earlier work	[MODEL], not the Einstein–Hilbert flow

What the paper does not claim. The Weyl-gauged QDG sector is absent from the derived flow; the fixed point is not shown to be gauge independent; its critical exponents are the exponents of this truncation, and the number of relevant directions is 2 at $n_f = 3$ in this truncation only; no robustness against the Weyl anomaly is claimed, since the anomaly coefficients do not enter the Einstein–Hilbert flow and the quantity that would enter a truncation with R^2 is ϱ , not computed here in the flow.

Open problems.

- Anomaly of the gauged Weyl symmetry of QDG, including graviton, dilaton, V_μ , $\tilde{a}_\mu$ and fermions, in a gauge-independent formulation; classification of $H^1(s|d)$ for $\text{Diff} \ltimes \text{Weyl}$ with dynamical V_μ .
- Computation of Σ_k for the truncations of Section 6, i.e. of the violation of (13), and a Weyl-covariant regulator.
- Contribution of V_μ , $\tilde{a}_\mu$ and ϕ , with their couplings to curvature, to η and β_λ ; mass dependence beyond the threshold factor $1/(1+w)$.
- Gauge parameters $\alpha \neq 1$ and the exponential parametrisation; the R^2 truncation with QDG-like content.
- The theory with $R(\Gamma)^2$ terms: new degrees of freedom and ghosts.

Falsifiability. Nothing in this paper predicts an observable. The results are structural: the BV–FRG formulation clarifies where the symmetry breaking of a regulator resides, which pieces of the distortion sector can matter for the anomaly, and which claims about a fixed point of QDG survive. QDG is a theoretical proposal.

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Conflicts of Interest

The author is the Editor-in-Chief of JAGP, the journal in which this article is published. This constitutes a structural conflict of interest. External anonymous peer review has been used to mitigate it. The author has recused himself from editorial handling.

Additional Information

During the preparation of this manuscript, the author used large language models for assistance with editing, language refinement, and verification of several derivations against previously established results. All scientific claims, equations, and conclusions are the author's own.

A Scripts

All numbers quoted were produced by the following scripts (numpy and scipy only; folder `code/`).

- `bvfrg_checks.py`: expansions (2), matrix N (6) and its nullities, absence of mixing between the irreducible parts, corrected variation (4), $C - A = 3(R - P)$, rank of the principal symbol, anomaly coefficients as exact fractions.
- `partIII_brst.py`: nilpotency in an exact graded algebra, antifield table, signs of $s\Gamma$ and $s\psi$ (Proposition 3.1).
- `partIII_anomaly.py`: heat-kernel coefficients (c, a, ρ) and the on-shell graviton number.
- `partI_II_audit_algebra.py`, `partI_II_audit_weyl.py`: the algebra of D and the Weyl identities of Proposition 2.2 (exact ranks, jets).
- `eh_flow_trace.py`, `fixed_point_cubic.py`, `partIV_solutions_check.py`: the flow (18), the cubic, the checks of the end of the branch.
- `bvfrg_flow.py`, `bvfrg_fixed_point.py`, `bvfrg_tolerance.py`: Tables of Section 6, mass dependence, $n_{\text{crit}}(w)$, tolerance to extra fields.
- `rre.py`, `rre_run50.py`, `bvfrg_rre_qdg.py`: regulator-robust existence along the QDG-like lines.
- `model_flow.py`, `bvfrg_model_flow_check.py`: the phenomenological flow.

The scripts `partI_II_*`, `partIII_*`, `eh_flow_trace.py`, `fixed_point_cubic.py`, `model_flow.py`, `rre*.py`, `partIV_solutions_check.py` and `jets.py` are those of the QDG monograph [2], re-run for this paper.

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