

Flow Equations for Tensorial Operators in Quantum Distortion Gravity, and the Open Problem of BRST Compatibility

L. E. Pavese^{*}¹

¹Independent Researcher, Italy

October 8, 2026

Abstract

We study the renormalization-group flow of a local composite operator $\mathcal{O}_I$ (a tensor built from the metric, the independent connection, the dilaton, the Weyl gauge field and the fermions of Quantum Distortion Gravity, QDG) by coupling it to an external source $\mathcal{J}$ and differentiating the Wetterich equation. The *response functional* $\mathcal{T}_I = \partial\Gamma_k/\partial\mathcal{J}|_{\mathcal{J}=0}$ is shown to be the one-point function of $\mathcal{O}_I$ in the regulated theory at fixed mean field, and it obeys a closed, linear, second-order functional equation, $\partial_t\mathcal{T}_I = -\frac{1}{2}\text{STr}[G_k\mathcal{T}_I^{(2)}G_k\partial_t R_k]$, whose only input beyond the propagator G_k is the Hessian of $\mathcal{T}_I$ itself. Under the stated hypotheses $\mathcal{T}_I$ transforms as a tensor of the type of $\mathcal{O}_I$ under background diffeomorphisms. The equation and its consequences are checked numerically in a two-variable model with an exactly solvable regulated integral. We then ask whether the flow is compatible with the BRST–BV structure of QDG. The answer is negative in the naive sense: a field-independent quadratic regulator is not BRST invariant, the Hessian of the BV action at fixed antifields is not antifield independent, and in the model the Ward identity is violated by an explicit, non-zero regulator term at every finite scale tested. For a BRST-closed integrated operator we derive the linearised modified Slavnov–Taylor identity that $\mathcal{T}_I$ obeys. Whether this regulator term can be controlled, or removed by a BRST-invariant regularisation, for the full QDG truncation is an open problem; the three-point input has not been computed, and the status of the Weyl anomaly of the gauged symmetry is open.

Keywords: quantum gravity, metric-affine gravity, Weyl symmetry, functional renormalization group, Batalin–Vilkovisky quantization, Slavnov–Taylor identities, composite operators

1 Introduction

Quantum Distortion Gravity (QDG) [1, 2] is a metric-affine theory with gauged Weyl symmetry: the affine connection $\Gamma^\rho_{\mu\nu}$ is an independent field, its difference from the Levi-Civita connection is the distortion $D^\rho_{\mu\nu}$, and local Weyl invariance is a gauge symmetry compensated by a dilaton ϕ and a gauge vector V_μ . Its renormalization-group (RG) behaviour is usually discussed through the flow of a few couplings. This note is about a complementary object: the flow of *tensorial composite operators*, such as the curvature, the stress tensor or a component of the distortion, whose scale dependence is not captured by the running of the couplings alone.

The note has two parts.

^{*}Corresponding author e-mail: luicapv02@gmail.com

Part one: the flow of a composite operator (Sections 2–4). Coupling $\mathcal{O}_I$ to a source and differentiating the Wetterich equation [4] gives an exact flow equation for the response functional $\mathcal{T}_I[\Phi]$ (Proposition 3.1). It is a standard application of functional-RG technique; composite-operator flows are treated in the general literature [7], and we make no claim of novelty for the derivation. What we add is (i) a precise definition and an elementary proof that $\mathcal{T}_I$ is the one-point function of $\mathcal{O}_I$ (Proposition 2.2), which is needed to interpret the equation, (ii) the observation that the three-point input of the flow is the Hessian of $\mathcal{T}_I$ itself, so the equation is closed and linear in $\mathcal{T}_I$ given the propagator, (iii) a covariance statement with explicit hypotheses (Proposition 3.3), and (iv) a numerical verification of all identities in a model where the regulated functional integral can be done by quadrature (Section 4).

Part two: BRST–BV compatibility (Section 5). QDG is a gauge theory with a BRST–BV formulation [2]. One may be tempted to argue that if the regulator R_k is independent of the fields, and the classical BV action satisfies the master equation, then the Wetterich flow preserves the master equation at every scale, and hence so does the flow of $\mathcal{T}_I$. This is not true, and we show why, in the general setting and in the model. The correct statement is the modified (“flowing”) Slavnov–Taylor identity [5, 6, 7, 8], which has an explicit regulator-dependent term. For a BRST-closed integrated operator we derive the linearised form of that identity satisfied by $\mathcal{T}_I$ (Proposition 5.1). The BV functional renormalization group of QDG, including the general modified identity and the status of the anomaly, is treated in a companion paper [3]; here we only need its structure.

Status labels. [STANDARD]: standard, source cited. [DERIVED]: derived in this paper. [NUMERICAL]: checked by a script in `code/` (numpy only). [ASSUMPTION]: stated assumption. [OPEN]: open problem. *Proposition* is proved under the stated hypotheses; *Remark* is not proved.

Conventions. The flow equation is formulated in Euclidean signature (as in the companion papers), with $\hbar = 1$, $t = \ln(k/k_0)$, and STr the supertrace over all field components, with a relative minus sign for Grassmann-odd fields and ghosts. Grassmann signs are otherwise suppressed in Section 5. The classical QDG conventions (signature $(-, +, +, +)$, Weyl weights, $D_\mu\phi = (\partial_\mu - V_\mu)\phi$) are those of [2].

2 The response functional

Let Φ^A denote all fields of the gauge-fixed theory (QDG: $g_{\mu\nu}$ or $h_{\mu\nu}$ on a background, the connection or its distortion, ϕ , V_μ , ψ , ghosts) and $S[\Phi]$ the gauge-fixed Euclidean action. Let $\mathcal{O}_I^{\mu_1\cdots\mu_n}(x)$ be a local operator, a function of Φ and its derivatives, and $\mathcal{J}_{\mu_1\cdots\mu_n}^I(x)$ a source that is independent of the fields. Let $\Delta S_k = \frac{1}{2}\Phi^A R_{k,AB}\Phi^B$ be the regulator term, with R_k independent of $\mathcal{J}$. Define

$$Z_k[j; \mathcal{J}] = \int D\Phi \exp \left\{ -S[\Phi] - \int \mathcal{J}^I \mathcal{O}_I - \Delta S_k[\Phi] + j_A \Phi^A \right\}, \quad W_k = \ln Z_k, \quad (1)$$

$\varphi^A = \delta W_k / \delta j_A$, and the effective average action of the theory with action $S + \mathcal{J} \cdot \mathcal{O}$,

$$\Gamma_k[\varphi; \mathcal{J}] = -W_k + j_A \varphi^A - \Delta S_k[\varphi]. \quad (2)$$

Throughout, φ is held fixed when $\mathcal{J}$ is varied (the source j adjusts).

Definition 2.1 (Response functional). $\mathcal{T}_{\mu_1\cdots\mu_n}^I[\varphi](x) \equiv \left. \frac{\delta \Gamma_k[\varphi; \mathcal{J}]}{\delta \mathcal{J}_{\mu_1\cdots\mu_n}^I(x)} \right|_{\mathcal{J}=0}$.

Proposition 2.2 ($\mathcal{T}$ is a one-point function). *[DERIVED]* $\mathcal{T}_{\mu_1 \dots \mu_n}^I[\varphi](x) = \langle \mathcal{O}_I^{\mu_1 \dots \mu_n}(x) \rangle_{k, \varphi}$, the expectation value of $\mathcal{O}_I$ in the regulated theory (1) at $\mathcal{J} = 0$ and with $j = j[\varphi]$ fixed by $\langle \Phi \rangle = \varphi$.

Proof. Differentiate (2) at fixed φ . Since $\partial W_k / \partial \mathcal{J}|_j = -\langle \mathcal{O} \rangle$ and $\partial W_k / \partial j_A = \varphi^A$,

$$\left. \frac{\partial \Gamma_k}{\partial \mathcal{J}} \right|_\varphi = - \left. \frac{\partial W_k}{\partial \mathcal{J}} \right|_j - \left(\frac{\partial W_k}{\partial j_A} - \varphi^A \right) \left. \frac{\partial j_A}{\partial \mathcal{J}} \right|_\varphi = \langle \mathcal{O} \rangle. \quad \square$$

□

At finite k this is not the classical value $\mathcal{O}[\varphi]$: modes above k are integrated out and contribute loop corrections. For $k \rightarrow 0$ it becomes the usual expectation value in the presence of the mean field φ (when the limit exists). The operator $\mathcal{O}$ needs a definition of its composite nature (renormalization of the product of fields at one point); the equation below holds for the regulated functional integral as written and does not by itself address that question.

3 The flow equation

Since R_k does not depend on $\mathcal{J}$, the Wetterich equation [4] holds for $\Gamma_k[\varphi; \mathcal{J}]$ at every $\mathcal{J}$,

$$\partial_t \Gamma_k[\varphi; \mathcal{J}] = \frac{1}{2} \text{STr}[G_k \partial_t R_k], \quad G_k = (\Gamma_k^{(2)}[\varphi; \mathcal{J}] + R_k)^{-1}, \quad (3)$$

with $\Gamma_{AB}^{(2)} = \delta^2 \Gamma_k / \delta \varphi^A \delta \varphi^B$.

Proposition 3.1 (Flow of the response functional). *[DERIVED]* Let $G_k = G_k[\varphi; 0]$. Then

$$\partial_t \mathcal{T}_{\mu_1 \dots \mu_n}^I[\varphi](x) = -\frac{1}{2} \text{STr} \left[G_k \frac{\delta^2 \mathcal{T}_{\mu_1 \dots \mu_n}^I(x)}{\delta \varphi \delta \varphi} G_k \partial_t R_k \right]. \quad (4)$$

Equivalently, with the three-point function $\Gamma_{AB;I}^{(3)}(y, z; x) \equiv \delta \Gamma_{AB}^{(2)}(y, z) / \delta \mathcal{J}^I(x)|_{\mathcal{J}=0}$, the right-hand side is $-\frac{1}{2} \text{STr}[G_k \Gamma_{;I}^{(3)}(x) G_k \partial_t R_k]$, and $\Gamma_{AB;I}^{(3)} = \delta^2 \mathcal{T}^I(x) / \delta \varphi^A \delta \varphi^B$.

Proof. Apply $\delta / \delta \mathcal{J}^I(x)$ at $\mathcal{J} = 0$ to (3). The derivative commutes with ∂_t and, because R_k is $\mathcal{J}$ -independent, acts only on G_k , with

$$\delta G_k = -G_k \delta \Gamma_k^{(2)} G_k.$$

Finally $\delta \Gamma_{AB}^{(2)} / \delta \mathcal{J} = \delta^2 (\delta \Gamma_k / \delta \mathcal{J}) / \delta \varphi^A \delta \varphi^B = \delta^2 \mathcal{T}^I / \delta \varphi^A \delta \varphi^B$, since functional derivatives with respect to φ and $\mathcal{J}$ commute. □

Remark 3.2 (Structure and consistency checks). *[DERIVED]* (i) Given the propagator G_k (that is, given $\Gamma_k^{(2)}$ of the truncation), (4) is a *linear* second-order functional differential equation for $\mathcal{T}^I$. The three-point function needed in the flow of a composite operator is not an independent input: it is the Hessian of $\mathcal{T}^I$. For the initial condition $\mathcal{T}_{k=\Lambda}^I = \mathcal{O}_I[\varphi]$ the first iteration is $\partial_t \mathcal{T}^I = -\frac{1}{2} \text{STr}[G_k (\delta^2 \mathcal{O}_I / \delta \varphi \delta \varphi) G_k \partial_t R_k]$, a one-loop expression whose vertex is the second variation of the classical local operator. (ii) If $\mathcal{O}_I = \Phi^A(x)$ itself, then $\Gamma_k[\varphi; \mathcal{J}] = \Gamma_k[\varphi] + \mathcal{J} \varphi$ exactly, $\mathcal{T} = \varphi$, $\mathcal{T}^{(2)} = 0$ and (4) is $0 = 0$. (iii) If $\mathcal{O}_I = \partial S / \partial c_i$ is the operator conjugate to a coupling c_i of the action, then $\mathcal{T} = \partial \Gamma_k / \partial c_i$ and (4) is the derivative of (3) with respect to c_i : the equation for the response functional of the operators conjugate to the couplings is the linearisation of the flow around a given trajectory (it contains the stability matrix of the couplings). (iv) For a free scalar with $\mathcal{O} = \varphi^2$, $\Gamma^{(2)} = p^2 + m^2$, (4) gives $\partial_t \langle \varphi^2 \rangle_k = - \int \frac{d^d p}{(2\pi)^d} \partial_t R_k / (p^2 + m^2 + R_k)^2$, the scale derivative of $\int \frac{d^d p}{(2\pi)^d} (p^2 + m^2 + R_k)^{-1}$.

Proposition 3.3 (Covariance). *[DERIVED] Suppose that the extended regulated action $S[\Phi; \bar{\Phi}] + \int \mathcal{J}^I \mathcal{O}_I + \Delta S_k[\Phi; \bar{\Phi}]$ (with $\bar{\Phi}$ the background fields on which the gauge fixing and the regulator are built) and the measure are invariant under a simultaneous diffeomorphism f acting on Φ , on $\bar{\Phi}$ and on $\mathcal{J}$ (the latter as a tensor of the type dual to $\mathcal{O}_I$), and that $\mathcal{O}_I$ is a tensor of type $(\mu_1 \dots \mu_n)$. Then $\mathcal{T}^I[f^* \varphi, f^* \bar{\Phi}] = f^* \mathcal{T}^I[\varphi, \bar{\Phi}]$, and so is $\partial_t \mathcal{T}^I$: both are tensor fields of the same type as $\mathcal{O}_I$.*

Proof. Under the hypotheses $\Gamma_k[f^* \varphi, f^* \bar{\Phi}; f^* \mathcal{J}] = \Gamma_k[\varphi, \bar{\Phi}; \mathcal{J}]$ (change of integration variable in (1), with j transformed accordingly). Differentiating with respect to $\mathcal{J}$ at $\mathcal{J} = 0$ gives the statement for $\mathcal{T}^I$, and ∂_t commutes with f^* . $\square$

The hypotheses are those of the background-field method: a background-covariant gauge fixing and a regulator built from the background-covariant Laplacian. This is the symmetry under which the background fields transform together with the fluctuations; it is not the quantum BRST symmetry, whose status is the subject of Section 5. The same argument gives the Lorentz covariance of $\mathcal{T}$ for fermionic operators if the frame is transformed too.

4 A model check

The identities of Propositions 2.2–3.1, and those of Section 5, can be checked exactly in a model with finitely many degrees of freedom, where the Wetterich equation holds as an identity for the regulated integral (1). [NUMERICAL] Take $x \in \mathbb{R}^2$,

$$S = \frac{1}{2} m^2 r^2 + \frac{1}{4} \lambda r^4, \quad \mathcal{O} = r^2 + \kappa r^4, \quad r^2 = x_1^2 + x_2^2, \quad R_k = k^2 \text{diag}(1, c_2),$$

with $m^2 = 1$, $\lambda = 0.7$, $\kappa = 0.3$. The action and the operator are invariant under rotations, generated by $\epsilon = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$; the regulator is invariant only for $c_2 = 1$. The integral (1), its Legendre transform and the derivatives were computed by two-dimensional trapezoid quadrature (spectrally accurate for these integrands) and finite differences (script `composite_toy.py`, output `composite_toy.out`). At $k = 1.3$, $\varphi = (0.45, -0.30)$:

identity	$c_2 = 3$	$c_2 = 1$
$\partial \Gamma / \partial \mathcal{J} = \langle \mathcal{O} \rangle$ (Prop. 2.2), difference	1×10^{-12}	2×10^{-11}
Wetterich equation, $\mathcal{J} = 0$ and 0.2, difference	$\leq 6 \times 10^{-10}$	$\leq 10^{-9}$
$\partial_t \mathcal{T} = -\frac{1}{2} \text{tr}[G \mathcal{T}'' G \partial_t R]$ (Prop. 3.1), difference	5×10^{-7}	5×10^{-7}
$\Gamma' \cdot \epsilon \varphi = \text{tr}(\epsilon^\top R_k G)$ (Eq. (6)), difference	3×10^{-16}	2×10^{-17}
$\nabla \mathcal{T} \cdot \epsilon \varphi = -\text{tr}(\epsilon^\top R_k G \mathcal{T}'' G)$ (Prop. 5.1), difference	9×10^{-8}	9×10^{-9}
value of $\Gamma' \cdot \epsilon \varphi$ (violation of the Ward identity)	+0.0200	0

The differences of the third and fifth rows are limited by the finite-difference step used for $\mathcal{T}''$. The last row is discussed in Section 5.

5 BRST–BV compatibility

5.1 What is derived in QDG and what is not

The classical BRST–BV structure of QDG is established in the QDG monograph [2] (script `partIII_brst.py`, re-run for this note, output `partIII_brst.out`): the BRST transformations

of $\text{Diff} \ltimes \text{Weyl}$ (with local Lorentz transformations when fermions are present) are nilpotent on all fields and ghosts; the BV action

$$S_{BV} = S_{\text{cl}}[\Phi] + \int \Phi_A^* R^A[\Phi], \quad R^A = s\Phi^A, \quad (5)$$

is linear in the antifields because R^A does not depend on them, and satisfies the classical master equation $(S_{BV}, S_{BV}) = 0$; in the unitary gauge $\phi = \phi_0$ the Weyl ghosts decouple and the ghost determinant is $\det \Delta_{\text{FP}} \times \det(\phi_0)$. [DERIVED] What is only formulated is the quantum master equation, whose obstruction is the anomaly of the gauged Weyl symmetry; its coefficients for QDG are not known [2, 3]. In what follows we do not use the anomaly, only the classical master equation.

5.2 Why compatibility is not automatic

Consider the claim: *if R_k is independent of the fields and $\Gamma_\Lambda = S_{BV}$ satisfies the master equation, then Γ_k satisfies it for all k .* It fails for the following reasons.

(a) The regulator breaks the symmetry. A quadratic regulator $\Delta S_k = \frac{1}{2} \Phi R_k \Phi$ has BRST variation $s\Delta S_k = R_{k,AB} \Phi^B R^A \neq 0$ in general, even if R_k is field independent: field independence of R_k says nothing about $R^A \partial_A \Delta S_k$. For Diff the transformations are inhomogeneous or bilinear in (c, Φ) , and for Weyl the fields carry weights while the regulator is built from a Weyl-non-covariant Laplacian.

(b) The Hessian depends on the antifields. $\Gamma_k^{(2)}$ is the Hessian with respect to the fields *at fixed antifields*. In (5), R^A is bilinear in a ghost and a field for $g_{\mu\nu}$, $\Gamma_{\mu\nu}^\rho$, ϕ , V_μ , ψ (for example $sg_{\mu\nu} = \mathcal{L}_c g_{\mu\nu} + 2c_W g_{\mu\nu}$), and quadratic in the ghosts for c^μ , c_W and λ^{ab} . Hence the mixed ghost–field blocks of $\delta^2 S_{BV} / \delta \Phi \delta \Phi$ equal $\Phi_A^* \partial^2 R^A / \partial \Phi \partial \Phi$, which is linear in the antifields and not zero. The statement that the propagator, or the flow, does not depend on Φ^* (“ $\Gamma_k^{(2)}$ does not depend explicitly on Φ^* ”) is therefore false for the BV action of QDG. The antifields may be treated as spectators in the flow (because R_k does not depend on them), but G_k depends on them parametrically.

(c) The model. In the model of Section 4 (rotation as the gauge symmetry, $R^A = \epsilon^A_B \Phi^B$ linear and divergence free) the identity obtained by integrating a total derivative is exact and elementary. From $0 = \int d^2x \partial_A (R^A e^{-S - \Delta S_k + j \cdot x})$, $R^A \partial_A S = 0$, $j_A = \Gamma_{k,A} + R_{k,AB} \varphi^B$ and the connected correlator $\langle \Phi^A \Phi^B \rangle_c = G_k^{AB}$, one finds

$$\Gamma_{k,A} \epsilon^A_B \varphi^B = \text{tr}(\epsilon^\top R_k G_k). \quad (6)$$

The right-hand side vanishes for a rotation-invariant regulator, but not for $c_2 \neq 1$: the numerical value is +0.0200 at $k = 1.3$, and it is positive for every k tested between 0.05 and 24 (Table 1; script `violation_range.py`), with a maximum near $k \approx 1.5$ and decreasing towards zero at both ends of the range ($\propto k^4$ at small and $\propto k^{-2}$ at large k , as read off from the output). Field independence of R_k is exact in this model, so it is not sufficient for preserving the symmetry. [NUMERICAL]

5.3 The modified identity and its linearisation for composite operators

The correct general statement is the modified Slavnov–Taylor identity (mSTI) for flows with a symmetry-breaking regulator [STANDARD] [5, 6, 7, 8]. In the notation of Section 2, with antifield sources Φ_A^* for the BRST variations added to (1) (term $\Phi_A^* R^A$ in the exponent), $\Gamma_k = \Gamma_k[\varphi, \Phi^*]$,

Table 1: Violation $\Gamma'_k \cdot \epsilon\varphi = \text{tr}(\epsilon^\top R_k G_k)$ of the Ward identity in the model, $c_2 = 3$, $\varphi = (0.45, -0.30)$ (scripts `composite_toy.py`, `violation_range.py`). It is zero for $c_2 = 1$ at all k .

k	0.1	0.5	1.0	2.0	4.0	8.0	16
violation	0.0004	0.0075	0.0178	0.0174	0.0069	0.0019	0.0005

$\Gamma_k^{*A} = \delta\Gamma_k/\delta\Phi_A^* = \langle R^A \rangle$, and for a BRST vector field R^A that is independent of the antifields and nilpotent, the argument of the companion paper [3] gives (Grassmann signs suppressed)

$$\Gamma_{k,A} \Gamma_k^{*A} = \mathcal{K}_k - \mathcal{I}, \quad \mathcal{K}_k = R_{k,AB} G_k^{BC} \frac{\delta^2 \Gamma_k}{\delta\Phi_A^* \delta\varphi^C}, \quad \mathcal{I} = \langle \partial_A R^A \rangle, \quad (7)$$

where $\frac{1}{2}(\Gamma_k, \Gamma_k) = \Gamma_{k,A} \Gamma_k^{*A}$. The term $\mathcal{I}$ does not involve the regulator (for the BRST vector fields of QDG it is the source of the Weyl anomaly, and it is linear in the ghosts); $\mathcal{K}_k$ is the regulator-induced violation. Eq. (6) is (7) in the model: there $\mathcal{I} = 0$, $\Gamma^{*A} = \epsilon^A_B \varphi^B$ and $\mathcal{K} = \text{tr}(\epsilon^\top R_k G_k)$. [NUMERICAL]

Now let $\mathcal{O}_I$ be such that its integral is BRST closed, $R^A \partial_A \int \mathcal{O}_I = 0$, and take the source $\mathcal{J}^I$ constant (so that the source term is itself BRST invariant). In QDG the operators conjugate to the couplings of the Weyl-invariant ultraviolet action ($\phi^2 \tilde{R}$, $(D\phi)^2$, ϕ^4 , $\phi^2 I_i[\tilde{D}]$, the field strengths, the Yukawa term) are of this kind, since the action is gauge invariant for all values of its couplings.

Proposition 5.1 (Linearised mSTI for composite operators). *[DERIVED] Let $\mathcal{O}$ be BRST closed in the above sense, let R^A be affine in the fields in the sense that $\partial_A R^A$ is linear in the ghosts, and write $\mathcal{T} = \mathcal{T}[\varphi, \Phi^*] = \partial\Gamma_k/\partial\mathcal{J}|_0$ for the response functional of the integrated operator. Then*

$$\Gamma_{k,A} \mathcal{T}^{*A} + \mathcal{T}_{,A} \Gamma_k^{*A} = R_{k,AB} \left[G_k^{BC} \frac{\delta^2 \mathcal{T}}{\delta\Phi_A^* \delta\varphi^C} - (G_k \mathcal{T}^{(2)} G_k)^{BC} \frac{\delta^2 \Gamma_k}{\delta\Phi_A^* \delta\varphi^C} \right]. \quad (8)$$

The left side is $\partial_{\mathcal{J}}[\frac{1}{2}(\Gamma_k, \Gamma_k)]_{\mathcal{J}=0}$. The identity is preserved by the flow (4).

Proof. The derivation of (7) uses (1) $R^A \partial_A S_\Psi = 0$, (2) $R^A \partial_A (\Phi_B^* R^B) = \Phi_B^* s^2 \Phi^B = 0$, and (3) the form of ΔS_k . Adding $\mathcal{J} \int \mathcal{O}$ to the action leaves (1)–(3) intact because $R^A \partial_A \int \mathcal{O} = 0$. So (7) holds for $\Gamma_k[\varphi, \Phi^*; \mathcal{J}]$ at every $\mathcal{J}$. Differentiate with respect to $\mathcal{J}$ at $\mathcal{J} = 0$: the left side gives that of (8); $\mathcal{I} = \langle \partial_A R^A \rangle$ is evaluated at the mean ghost fields and does not depend on $\mathcal{J}$; in $\mathcal{K}_k$ the $\mathcal{J}$ -derivative acts on G_k (giving $-G_k \mathcal{T}^{(2)} G_k$, as in Proposition 3.1) and on $\delta^2 \Gamma_k / \delta\Phi^* \delta\varphi$ (giving $\delta^2 \mathcal{T} / \delta\Phi^* \delta\varphi$). Since both (3) and (7) follow from the same regulated integral, a solution of the flow that satisfies (7) at the initial scale satisfies it at all scales. $\square$

In the model the term with $\delta^2 \mathcal{T} / \delta\Phi^* \delta\varphi$ is absent ($\mathcal{T}$ does not depend on antifields), and (8) reduces to $\nabla \mathcal{T} \cdot \epsilon\varphi = -\text{tr}(\epsilon^\top R_k G_k \mathcal{T}'' G_k)$, which is the fifth row of the table of Section 4. [NUMERICAL]

Remark 5.2 (What this says). The response functional of a gauge-invariant operator is itself BRST invariant only up to the regulator term on the right of (8). This term is built from the same objects as the flow (propagators, the Hessian of $\mathcal{T}$, and the regulator) and it vanishes if the regulator is BRST invariant, and in the limits $k \rightarrow 0$ and $k \rightarrow \infty$, where $R_k G_k \rightarrow 0$ or $\rightarrow 1$ and the traces above vanish (in the model, because $\text{tr} \epsilon = 0$). For a general truncation of the flow the violation is not zero and its size measures the error of the truncation; for QDG it has not been computed. Whether a regularisation exists for which it vanishes by construction (for instance a background-field gauge with a Weyl- and diffeomorphism-covariant cutoff) is open.

6 Specialisation to QDG: what remains to be done

Field content of the supertrace. In the infrared theory the bosonic content is the graviton (2 polarisations), the massive Weyl vector $\hat{V}_\mu$ (3) and the massive axial vector $\hat{a}_\mu$ (3), plus n_f Dirac fermions; the remaining 60 components of the distortion are auxiliary; in the unitary gauge the Weyl ghosts decouple and only the diffeomorphism ghosts remain [2]. The counting $2 + 3 + 3 = 8$ is a classical count at quadratic order. In the supertrace of (4) the fermions and ghosts enter with a relative minus sign.

Input required. For a given operator $\mathcal{O}_I$ (the curvature, the stress tensor, a component of the distortion), (4) needs (i) the propagator G_k of the truncation, that is the Hessian $\Gamma_k^{(2)}$ in the chosen gauge, and (ii) the second variation $\delta^2 \mathcal{O}_I / \delta \varphi \delta \varphi$ for the first iteration. Even for the graviton sector alone the vertex structure of Einstein–Hilbert gravity in a de Donder-type gauge is lengthy, and the full QDG content (Proca, axial, distortion couplings β_i , g_V , g_a , Yukawa coupling) is much larger. This is a well-defined but substantial computation, suited to tensor-algebra software, and is not attempted here. The Einstein–Hilbert flow with minimally coupled matter used in the companion papers is a projection of (3) on two couplings in a particular gauge; it does not include the Weyl-gauged sector, and it does not impose (7) [3].

7 Open problems

Open Problem 7.1 (Size of the regulator term). [OPEN] Compute $\mathcal{K}_k$ of (7), and its linearisation (8) for the operators conjugate to the QDG couplings, for the QDG truncation, instead of assuming it vanishes; or identify a BRST-invariant regularisation (for instance a background-field gauge with a covariant cutoff) for which it vanishes by construction.

Open Problem 7.2 (Anomaly of the gauged Weyl symmetry). [OPEN] The term $\mathcal{I}$ in (7) and in the quantum master equation is the obstruction to Weyl invariance of the measure. Its coefficients for the gauged symmetry of QDG, coupled to the dynamical V_μ , ϕ and Γ , are not known; the conformal-anomaly coefficients of the matter fields on a fixed background are a different quantity [2, 3].

Open Problem 7.3 (Explicit solution of (4) for QDG). [OPEN] Compute the propagator G_k and the first iterate of (4) for a gauge-invariant local operator of QDG, with the fermion and ghost signs, and compare with the flow of the couplings of the companion papers.

Open Problem 7.4 (Local sources and Weyl weight). [OPEN] Proposition 5.1 is stated for integrated operators with constant sources. For a local tensorial source the source must transform under the diffeomorphisms (and under Weyl rescalings, with the weight dual to that of $\mathcal{O}_I$), and its BRST variation then enters the identity through $\langle (s\mathcal{J})\mathcal{O} \rangle$. This extension has not been done.

8 Conclusions

The flow of a tensorial composite operator follows from the Wetterich equation in a closed linear form, Eq. (4), whose vertex is the Hessian of the operator’s own response functional; it is covariant under background diffeomorphisms under stated hypotheses, and it was verified numerically in a two-variable model. Compatibility of this flow with the BRST–BV structure of QDG is *not* automatic. A field-independent quadratic regulator is not BRST invariant, the Hessian of the BV action depends on the antifields, and in the model the Ward identity is violated by a definite, non-zero regulator term at every finite k tested. For a gauge-invariant integrated operator the response functional satisfies the linearised modified Slavnov–Taylor identity (8).

Computing the regulator term for the QDG truncation, and the anomaly of the gauged Weyl symmetry, are open problems; nothing in this note bears on the existence of a fixed point of QDG.

Acknowledgements. The author thanks the community for discussions and feedback. This work was conducted without institutional or financial support.

References

- [1] L. E. Pavesi, *On the Quantum Distortion Gravity*, Journal of Advanced Gravitational Physics **1**, 2 (2026).
- [2] L. E. Pavesi, *Quantum Distortion Gravity*, monograph, corrected second edition (2026).
- [3] L. E. Pavesi, *Batalin–Vilkovisky functional renormalization group for Quantum Distortion Gravity: regulator-induced symmetry breaking, anomaly status and fixed-point existence*, companion paper, Journal of Advanced Gravitational Physics (2026).
- [4] C. Wetterich, *Exact evolution equation for the effective potential*, Physics Letters B **301** (1993) 90–94.
- [5] U. Ellwanger, *Flow equations and BRS invariance for Yang–Mills theories*, Physics Letters B (1994), DOI 10.1016/0370-2693(94)90365-4, arXiv:hep-th/9402077.
- [6] M. Reuter, C. Wetterich, *Effective average action for gauge theories and exact evolution equations*, Nuclear Physics B **417** (1994) 181–214.
- [7] J. M. Pawłowski, *Aspects of the functional renormalisation group*, Annals of Physics (2007), DOI 10.1016/j.aop.2007.01.007, arXiv:hep-th/0512261.
- [8] Y. Igarashi, K. Itoh, T. R. Morris, *BRST in the exact RG*, Progress of Theoretical and Experimental Physics (2019), DOI 10.1093/ptep/ptz099, arXiv:1904.08231 [hep-th].