

Regulator Dependence of the Asymptotic-Safety Fixed Point in the R^2 Truncation with Standard-Model Matter Content

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Abstract

We study how the non-Gaussian fixed point of the Lauscher–Reuter R^2 truncation (gauge $\alpha = 1$, background S^4) depends on the shape of the regulator, for pure gravity and for minimally coupled matter with the field content of the Standard Model. The regulator is the exponential family $R^{(0)}(y) = sy/(e^{sy} - 1)$, which has $R^{(0)}(0) = 1$ for all s . Our implementation reproduces the analytic relations of Lauscher and Reuter to 5×10^{-7} and their fixed point and exponents for pure gravity. In pure gravity the fixed point, with three relevant directions and a complex pair of exponents, is found at all sampled $s \in [1, 2.5]$. With Standard-Model content ($N_S = 4$, $N_D = 22.5$, $N_V = 12$) there is a fixed point with three real positive exponents for $s > s_{\text{fold}} = 1.07210$; it is at large couplings ($g_* \simeq 1.7\text{--}3.9$, $\lambda_* \simeq -6$ to -22) and it disappears at s_{fold} in a saddle-node, where the leading exponent is of order 4×10^3 because the anomalous dimensions are near a pole. The values 1.0344 and 1.0722 reported in an earlier version as “edges” where the leading exponent diverges were produced by a finite-difference artefact and are withdrawn; for $N_D = 27$ there is no fold, only a run-away with $s > 1.083$. At the Einstein–Hilbert level the same content is regulator-robust, but at extreme couplings; the R^2 truncation confirms existence of a fixed point at $s \geq 1.5$ and does not confirm it at $s = 1$. We do not claim that the Standard Model is asymptotically safe.

Keywords: quantum gravity, asymptotic safety, functional renormalization group, R^2 truncation, regulator dependence, Standard Model

1 Introduction

Asymptotic safety [1] asks whether the renormalization-group (RG) flow of gravity defined by the Wetterich equation [2] has a non-Gaussian ultraviolet fixed point. Evidence in the Einstein–Hilbert (EH) truncation goes back to Reuter [3]; the next step in the derivative expansion, the addition of $\bar{\beta}_k R^2$, was carried out by Lauscher and Reuter [4] (LR) and extended in [5, 6, 7]. The effect of matter on the EH fixed point is discussed in [8, 9].

Position and exponents of a fixed point depend on the regulator, the gauge and the truncation. A weaker property that one hopes to be independent of the regulator is existence: regulator-robust existence (RRE), defined in the second edition of the book [12] (chapter on regulator-robust existence) for a finite set of regulators. At the EH level, with nine regulators (optimised profile with $a = 0.5, 0.75, 1, 1.5, 2, 3$ and exponential profile with $a = 0.5, 1, 2$), the book classifies 50 matter contents as robust (28), fragile (13) or absent (9) [12]. The Standard Model content ($X = 1$, $Y = -62$ in the notation of Section 6) is robust, but the fixed point is far

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from the origin: $(g_*, \lambda_*) = (3.2, -4.4)$ for the optimised cutoff with $a = 1$, and up to $g_* = 4.3$, $\lambda_* = -13.8$ over the nine regulators (reproduced in Appendix A). It is therefore outside the conventional range $g_* < 2$, $\lambda_* > -1$, which is a convention and not a derived bound.

The R^2 truncation is the natural extra check of that EH result: it adds the coupling $\beta = \bar{\beta}_k$ and the fourth-order propagator of the conformal mode, and the position of the fixed point inside the EH truncation is exactly where one fears that higher-order operators matter. This paper performs that check on a one-parameter regulator family, with the following outcome.

1. Our implementation of the LR flow, with a corrected linear system for the anomalous dimensions and one corrected sign of the printed appendix (Section 4), reproduces the analytic relations of LR and their pure-gravity fixed point and exponents. The item left open in the earlier version (the signs of the $\delta_{d,4}$ terms) is resolved by an internal-consistency derivation (Section 4).
2. In pure gravity the fixed point persists on the sampled values of $s \in [1, 2.5]$ (Section 5).
3. For Standard-Model content, there is a fixed point with three real, positive exponents for $s > s_{\text{fold}}$; the lower end is a saddle-node, not a divergence of an exponent (Section 7). The numbers $s_{\text{edge}} = 1.0722, 1.0344, 1.0833$ of the earlier version were an artefact of a finite-difference step (Section 7.3).
4. At $s = 1$, the member of the family that coincides with the exponential EH regulator with $a = 1$, we find no fixed point with three relevant directions for Standard-Model content. This is a statement about the branch that we can follow; we cannot exclude other zeros (Section 7.7).

Throughout, statements are labelled as *verified* (derived here or reproduced by a script of the accompanying code), *quoted* (from a source that was checked), or *hypothesis/open*. Section 8 collects them. The paper does not address the Weyl-gauged distortion sector of Quantum Distortion Gravity (QDG) [10, 11]: its fields enter neither the EH-level flow of the book nor the flow used here, and whether they modify it is an open problem.

2 The truncation and its flow

2.1 Truncation

LR [4] (their eqs. (4.1)–(4.2)) approximate the effective average action on the Euclidean metric by

$$\Gamma_k = 2\kappa^2 Z_N \int d^4x \sqrt{g} (-R + 2\bar{\lambda}_k) + \bar{\beta}_k \int d^4x \sqrt{g} R^2 + \Gamma_k^{\text{gf}} + \Gamma_k^{\text{gh}}, \quad (1)$$

with $\kappa^2 = (32\pi\bar{G})^{-1}$, a harmonic gauge fixing with $\alpha = 1$, a linear split of the metric, the cutoff of type B, and the flow evaluated on S^4 . The dimensionless couplings are

$$\lambda_k = k^{-2}\bar{\lambda}_k, \quad g_k = k^2 G_k, \quad \beta_k = \bar{\beta}_k k^{4-d} \quad (= \bar{\beta}_k \text{ in } d=4), \quad (2)$$

and $\eta_N = -\partial_t \ln Z_N$, $\eta_\beta = -\partial_t \ln \bar{\beta}_k$, $t = \ln k$. In $d=4$

$$\beta_\lambda = A_1 + \eta_N A_2 + \eta_\beta A_3, \quad \beta_g = (2 + \eta_N) g, \quad \beta_\beta = -\eta_\beta \beta, \quad (3)$$

where A_i, B_i, C_i are the coefficients of LR's appendices (A32)–(A35) and (B1)–(B9), functions of (λ, g, β) built from the threshold functions below. The anomalous dimensions solve the linear system (A34)–(A35) of [4],

$$\eta_N(1 - gB_2) - gB_3 \eta_\beta = gB_1, \quad C_2 \eta_N + (\beta + C_3) \eta_\beta = -C_1, \quad (4)$$

whose solution is

$$\eta_N = \frac{g[B_1(\beta + C_3) - C_1 B_3]}{D}, \quad \eta_\beta = -\frac{C_1(1 - gB_2) + gB_1 C_2}{D}, \quad D = (1 - gB_2)(\beta + C_3) + gC_2 B_3. \quad (5)$$

(The system written in the earlier version of this paper had inconsistent signs; the solution it quoted is (5), which is what the code uses.) The anomalous dimensions have a pole where $D = 0$. The proximity of the fixed point to such a pole is a diagnostic used below (the “pole distance”: the relative change of β at fixed (λ, g) needed to reach $D = 0$).

2.2 Threshold functions

With $R \equiv R^{(0)}(y)$ and $R' = dR^{(0)}/dy$, LR define (their (A29)–(A31))

$$\Phi_n^p(w) = \frac{1}{\Gamma(n)} \int_0^\infty dy y^{n-1} \frac{R - yR'}{(y + R + w)^p}, \quad \tilde{\Phi}_n^p(w) = \frac{1}{\Gamma(n)} \int_0^\infty dy y^{n-1} \frac{R}{(y + R + w)^p}, \quad (6)$$

$$\Psi_{n;m}^{p;q}(v, w) = \frac{(-1)^i}{\Gamma(n+i)} \int_0^\infty dy \frac{d^i}{dy^i} [(y + R)^m (R - yR')] \frac{y^{n+i-1}}{(y + R + w)^p \arg(y)^q}, \quad (7)$$

and $\tilde{\Psi}_{n;m;l}^{p;q}$ analogously with $(y + R)^m (2y + R)^l R$ in place of $(y + R)^m (R - yR')$; $i = 0$ for $n > 0$ and $i = 1$ for $n = 0$. The quantity

$$\arg(y) = 32\pi g\beta (y + R)^2 - \frac{1}{3} (y + R - 2\lambda) \quad (8)$$

is the denominator of the conformal-mode propagator in $d = 4$ at $P_k = k^2(y + R)$ (it equals A_2 at that argument). The functions with $n = 0$ reduce, for $R^{(0)}(0) = 1$, to boundary values: $\Phi_0^p(w) = \tilde{\Phi}_0^p(w) = (1 + w)^{-p}$ and $\Psi_{0;m}^{p;q}(v, w) = [(1 + w)^p \arg(0)^q]^{-1}$. They are evaluated here by Gauss–Legendre quadrature with N_G nodes on each of three segments $[0, 2/s]$, $[2/s, 8/s]$, $[8/s, 40/s]$.

Definition 2.1 (admissible fixed point). A zero $(\lambda_*, g_*, \beta_*)$ of (3) with $g_* > 0$, $\beta_* > 0$ is *admissible* if $\arg_{\min} \equiv \min_{y \geq 0} \arg(y) > 0$ at the fixed point and if the stability matrix $M = \partial(\beta_\lambda, \beta_g, \beta_\beta)/\partial(\lambda, g, \beta)$ has at least three eigenvalues with $\text{Re } \theta_i > 0$, θ_i being the eigenvalues of $-M$.

3 Regulator family

The $\delta_{d,4}$ terms of LR’s (B7)–(B9) assume $R^{(0)}(0) = 1$. We therefore use

$$R^{(0)}(y) = \frac{sy}{e^{sy} - 1}, \quad R^{(0)}(0) = 1 \quad \forall s > 0, \quad (9)$$

which is LR’s (3.11). LR scan $1 \leq s \leq 30$ in pure gravity and restrict to $s \geq 1$ because some threshold functions diverge for $s < 1$ in $d = 4$ [4]. We follow them and use $s \geq 1$. A larger s means a faster decay of $R^{(0)}$. Since s is a continuous parameter, no fraction ρ of regulators is defined; we report regions in s . The member $s = 1$ is the profile $z/(e^z - 1)$, i.e. the exponential EH regulator with $a = 1$ of the nine-regulator family of the book. The optimised profile of the EH analysis has $R^{(0)}(0) = a$ and is not part of this family (only $a = 1$ would satisfy the normalisation); nothing is said here about it.

4 Validation against Lauscher–Reuter

Analytic relations (verified). Table 1 compares our threshold functions and coefficients with relations of LR that do not require solving the flow: (5.8), (5.12), (5.13), (5.18) of [4]. The pure-gravity fixed point at $s = 1$ and its exponents are

$$(\lambda_*, g_*, \beta_*) = (0.3303, 0.2918, 0.00545), \quad \theta = \{28.82, 2.146 \pm 3.791 i\}, \quad (10)$$

to be compared with LR’s $(0.330, 0.292, 0.005)$ and $\theta = 2.15 \pm 3.79 i$, 28.8 (their (5.33), (5.35)). The largest deviation, 5×10^{-7} , is in (5.12). The old statement “agreement at 10^{-7} ” is therefore

relation	computed	LR	rel. deviation
(5.8) $\gamma_4 = C_1(0, 0, \beta)$	0.0024568042	0.0024568042	5×10^{-16}
(5.12) $\nu_4 d = \lim A_1/g$	0.76525285 [0.15915492]	0.76525319 [0.15915494]	4.5×10^{-7} [1.5×10^{-7}]
(5.13) $\varsigma_4 = \partial_\lambda C_1 _0$	0.026802379	0.026802379	6×10^{-10}
(5.18) $\omega_4 = -B_1(0)/(d-2)$	1.331386	1.331386	$< 10^{-15}$
fixed point, $s = 1$, θ	as above	as above	$< 10^{-3}$

Table 1: Validation against [4]; script `rre_R2_all.py quick` (output `out/quick_v6.txt`). Unbracketed values: exponential profile with $s = 1$. Bracketed values: Litim profile with $a = 1$. “Computed” is our evaluation of the left-hand side, “LR” the closed form of the right-hand side.

replaced by “ $\leq 5 \times 10^{-7}$ ”.

The signs of the $\delta_{d,4}$ terms (item resolved by internal consistency). In $d = 4$ the coefficients (B7)–(B9) contain terms $\propto \delta_{d,4}$. A sign discrepancy between our code and LR’s formulas had been left open. Script `check_LR_signs.py` compares three objects: (i) the bracket of the printed eq. (A28); (ii) the same bracket rebuilt from the $R^{(0)}$ -order traces of (A10) with the $\delta_{d,4}$ parts of the heat-kernel coefficients (A13), (A14) and of the primed traces (A23)–(A25); (iii) (B7)–(B9) as printed. At six random points $(\lambda, x = g\beta)$ the three agree to 3×10^{-14} (output `out/check_LR_signs.txt`). The remaining signs were then tested through their effect on the published numbers: of the seven other choices of the three signs, one has no fixed point near LR’s, and the others change $\text{Im } \theta_\pm$ from 3.79 to 3.84–4.08 and θ_3 from 28.82 to 25.85–29.07, while LR give 3.79 and 28.8. Only the printed signs reproduce both to the quoted precision. This establishes that the printed (B7)–(B9) are internally consistent with (A10), (A28) and with the published exponents. It is *not* an independent derivation of (A10) from the Wetterich equation, and the earlier discrepancy of our script is attributed to a sign convention in the code, not to LR.

One misprint (verified). The printed (B2) carries $+h_1 \tilde{\Psi}$ in A_2 . By (A26) the λ -flow contains $-h_1 Q[A_2^{-1} \mathcal{N}]$ and by (A32) $Q[\cdot \mathcal{N}] = \Psi - \frac{\eta_N}{2} \tilde{\Psi}$, so A_2 must contain $-h_1 \tilde{\Psi}$ (as A_1 contains $-h_1 \Psi$). With the printed sign the LR fixed point is not found; with the corrected sign it is reproduced (`out/check_LR_signs.txt`). The h_i are computed in exact rational arithmetic in $d = 4$; the values obtained ($h_1 = 2/3$, $h_{44} = 79/6$, $h_8 = 8$, $h_{36} = 14.4583$, ...) are those of the regression table of the code, taken from LR’s table D2.

5 Pure gravity

Table 2 gives the pure-gravity fixed point across the family ($N_G = 240$; the values at $N_G = 60$ are identical to the digits shown). Three relevant directions and the complex pair persist; $\text{Im } \theta_\pm$ decreases monotonically with s and is not linear in s . This is a re-examination of a known result with a new regulator scan, not a new result.

s	1.0	1.25	1.5	1.75	2.0	2.25	2.5
λ_*	0.3303	0.2989	0.2736	0.2528	0.2354	0.2207	0.2080
g_*	0.2918	0.3400	0.3851	0.4279	0.4689	0.5084	0.5466
$\text{Re } \theta_\pm$	2.146	2.152	2.161	2.174	2.187	2.202	2.217
$\text{Im } \theta_\pm$	3.791	3.573	3.432	3.334	3.264	3.213	3.175
θ_3	28.82	25.81	23.43	21.37	19.54	17.92	16.51
$\arg_{\min}$	0.0464	0.0427	0.0410	0.0408	0.0418	0.0425	0.0418

Table 2: Pure gravity, exponential family (`pure_gravity_scan.py`). The fit $\text{Im } \theta_\pm \approx 3.79 - 0.41(s - 1)$ of the earlier version is withdrawn: it is not linear.

6 Matter

Contributions. For N_S real scalars, N_D Dirac fermions and N_V massless vectors (minimally coupled, cutoff of type B, matter without anomalous dimensions) we use

$$X = N_S + 2N_D - 4N_V, \quad Y = N_S - 4N_D + 2N_V, \quad Z = \frac{29N_S - 11N_D - 62N_V}{2160}, \quad (11)$$

$$A_1 \rightarrow A_1 + \frac{2}{4\pi} g Y \Phi_2^1(0), \quad B_1 \rightarrow B_1 + \frac{4}{6 \cdot 4\pi} X \Phi_1^1(0), \quad C_1 \rightarrow C_1 + \frac{1}{(4\pi)^2} Z \Phi_0^1(0). \quad (12)$$

X, Y are the combinations of the EH-level analysis of the book. *Verified:* for the Litim profile with $a = 1$ ($\Phi_1^1(0) = 1$, $\Phi_2^1(0) = \frac{1}{2}$) the first two contributions are $\Delta B_1 = X/(6\pi)$ and $\Delta A_1 = gY/(4\pi)$, i.e. the EH-level matter terms $\Delta\eta \propto gX/(6\pi)$ and $\Delta\beta_\lambda = gY/(4\pi)$ of the book; the normalisation is therefore consistent with the EH flow.

The coefficient Z (verified as a_2 on S^4 ; identification with R^2 is a truncation assumption). The earlier version quoted Z without derivation. Script `check_heatkernel_S4.py` computes, from the exact spectra on the unit S^4 , the heat-kernel coefficients: for the scalar Laplacian $a_0 = 1$, $a_1 = \frac{1}{6}$ (in units of $R = 12$) and $2160 a_2 = 29$; for the squared Dirac operator per component $2160 a_2 = 11$ with $a_1 = -\frac{1}{3}$; for the Hodge–de Rham vector Laplacian $2160 a_2 = -4$; with two ghost scalars the vector sector gives $-4 - 2 \cdot 29 = -62$ (output `out/check_heatkernel_S4.txt`). This reproduces the three coefficients of Z . What this does *not* establish is that the whole on-sphere a_2 should be attributed to the R^2 operator: on S^4 the invariants R^2 , $R_{\mu\nu}R^{\mu\nu}$, $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ are not distinguished. This is the same projection LR use for the gravitational part, and it is a limitation of the truncation (hypothesis/open). We also do not treat the ghost zero modes on S^4 .

Sensitivity to Z (verified). Z enters only C_1 . Script `zsens.py` rescales $Z \rightarrow z Z$ for the Standard-Model fixed point (Table 3); even $z = 0$ moves λ_* by 6%, $\theta_{2,3}$ by $\leq 4\%$ and θ_1 by 18% at $s = 1.5$, and the lower end s_{fold} by 3×10^{-4} (1.071828 for $z = 0$ against 1.072104, $N_G = 480$). Therefore the unresolved attribution of a_2 to R^2 does not affect any qualitative statement below.

7 Standard-Model content

Throughout this section $N_S = 4$, $N_V = 12$ and $N_D = 22.5$ unless stated ($N_D = 22.5$ Dirac fermions represent 45 Weyl fermions, i.e. three generations; $X = 1$, $Y = -62$).

7.1 The fixed point along the family

Starting from the pure-gravity fixed point and switching on matter by continuation (script `r2_contents.py`) one does *not* reach a fixed point for the Standard-Model content: the branch of

s	z	$(\lambda_*, g_*, \beta_*)$	θ_1	θ_2, θ_3
1.5	0	(-7.215, 1.868, 0.03066)	12.62	3.622, 1.835
1.5	1	(-7.646, 1.978, 0.03030)	15.32	3.643, 1.901
1.5	2	(-8.088, 2.091, 0.02995)	18.46	3.659, 1.956
2.0	0	(-6.169, 2.262, 0.02662)	6.852	3.452, 1.570
2.0	1	(-6.761, 2.478, 0.02586)	8.144	3.514, 1.665
2.0	2	(-7.423, 2.719, 0.02522)	9.745	3.558, 1.740

Table 3: Sensitivity of the Standard-Model fixed point to the matter a_2 coefficient, $N_G = 240$.

the Reuter fixed point deformed by matter (called G below) ends before $N_D \simeq 8.9$ (Section 7.5). A fixed point of Standard-Model type (branch S) exists nevertheless: it was located with the earlier scripts, it is a converged zero of the flow, and a Newton search from a grid of seeds at $s = 1.5$ finds it independently (Section 7.7); it is then tracked in s (script `sm_branch.py`, $N_G = 240$; Table 4). Its three exponents are real and positive, there is no oscillatory approach, and $\arg_{\min} > 0$. The s values 1.5 and 2 reproduce the exponents quoted in the earlier version

s	λ_*	g_*	β_*	$\arg_{\min}$	$\theta_1, \theta_2, \theta_3$	pole distance
2.5	-6.04	2.910	0.02389	2.226	6.558, 3.399, 1.551	-2.0×10^{-1}
2.0	-6.76	2.478	0.02586	1.601	8.144, 3.514, 1.665	-1.6×10^{-1}
1.5	-7.65	1.978	0.03028	0.591	15.32, 3.643, 1.901	-7.2×10^{-2}
1.3	-8.14	1.773	0.03376	0.261	30.9, 3.691, 2.069	-3.3×10^{-2}
1.2	-8.60	1.703	0.03621	0.133	61.4, 3.713, 2.167	-1.7×10^{-2}
1.1	-10.53	1.874	0.03920	0.0353	310, 3.740, 2.135	-3.5×10^{-3}
1.075	-15.01	2.587	0.03981	0.0126	1493, 3.772, 1.451	-8.5×10^{-4}
<i>fold</i> 1.0721	-22.48	3.844	0.03966	0.0073	4490, 3.801, ≈ 0	-3.1×10^{-4}

Table 4: The Standard-Model-type fixed point along the family, $N_G = 240$ (the last line from $N_G = 960$). “Pole distance” is the relative change of β (at fixed λ_*, g_*) at which D of (5) vanishes. The stability matrix is computed with the Richardson stencil of Section 7.3.

($\{15.32, 3.64, 1.90\}$ and $\{8.14, 3.51, 1.67\}$).

7.2 The lower end: a saddle-node

Tracking in s alone stops near $s \simeq 1.075$ because $d\lambda_*/ds$ grows. We therefore take λ_* as the parameter and solve for (g_*, β_*, s) (script `fold_min.py`). Along the curve $s(\lambda_*)$ has a minimum s_{fold} : at fixed s the fixed point annihilates with a partner (saddle-node), $ds/d\lambda_* = 0$, and the smallest real exponent crosses zero there. Near the minimum $s(\lambda_*)$ is parabolic with a very small curvature ($d^2s/d\lambda_*^2 \simeq 3 \times 10^{-5}$ for $N_D = 22.5$), so s_{fold} is determined accurately (quadratic fit; residual 4×10^{-9}) whereas λ_*^{fold} and g_*^{fold} are determined only to about 10^{-2} relative. We therefore quote

$$s_{\text{fold}}(N_D = 15) = 1.03097, \quad s_{\text{fold}}(N_D = 22.5) = 1.07210 \quad (N_G \geq 480). \quad (13)$$

Verified facts about the fold: (i) $\arg_{\min}$ is positive, small (0.005–0.007) and attained at $y = 0$; (ii) θ_1 is large ($\sim 10^3$) but finite: it grows as the fixed point approaches the pole of η , with the pole at a relative distance 3×10^{-4} ($N_D = 22.5$) in β ; (iii) the fold position is almost independent of the matter a_2 term (3×10^{-4} , Section 6). *Interpretation (hypothesis):* the fixed point is destroyed by the nearby singularity of the anomalous dimensions and of the conformal-mode propagator; with $\arg_{\min} \approx 5\text{--}7 \times 10^{-3}$ and $\theta_1 \sim 4 \times 10^3$ the truncation cannot be trusted anywhere near the fold, so s_{fold} is a property of the truncation, not an estimate of where a fixed point of the full theory disappears.

N_D	N_G	s_{fold}	λ_*	g_*	$\arg_{\min}$	θ_1, θ_2	pole dist.
15	240	1.030969	-3.806	1.2164	0.00502	857, 3.510	-1.3×10^{-3}
15	480	1.030969	-3.806	1.2164	0.00502	857, 3.510	-1.3×10^{-3}
22.5	240	1.072091	-22.71	3.883	0.00725	4656, 3.801	-2.8×10^{-4}
22.5	480	1.072104	-22.48	3.844	0.00732	4491, 3.801	-3.1×10^{-4}
22.5	960	1.072104	-22.48	3.844	0.00732	4491, 3.801	-3.1×10^{-4}

Table 5: Lower end of the branch with three relevant directions, $N_S = 4$, $N_V = 12$. The third exponent is zero to the accuracy of the finite-difference stencil ($|\theta_3| \leq 10^{-2}$). s_{fold} is converged to 10^{-11} between $N_G = 480$ and 960 ; $N_G = 240$ differs by 1.3×10^{-5} and $N_G = 60$ was found insufficient for this branch with the earlier protocol and was not re-run here. Script `fold_min.py`, outputs `out/foldmin_*.json`.

7.3 Why the earlier “edges” are withdrawn

The earlier version located a lower edge as the point where the leading exponent “diverges and changes sign”, using central differences with step $h = 10^{-5}$ for the stability matrix. That protocol (kept as `edge_scan_old_h1e-5.py`; outputs in `out/edge_old_protocol/`) reproduces its own numbers: $s = 1.034443$ ($N_D = 15$, $N_G = 960$, $\lambda = -4.698$, $g = 1.487$) and $s = 1.072193$ ($N_D = 22.5$, $N_G = 960$, $\lambda = -25.15$, $g = 4.297$). Script `artifact_hscan.py` analyses the first point: at fixed (λ, g) , η_N and η_β have a pole in β at a relative distance of order 4×10^{-4} (at $\delta\beta \simeq -1.0 \times 10^{-5}$ for $\beta_* = 0.0232$), which is of the order of the finite-difference step. With $h = 10^{-5}$ the stencil straddles the pole and gives $\theta_1 \sim -2 \times 10^8$; for $h \leq 10^{-6}$ the exponents converge to (2720.6, 3.559, -2.94) (output `out/artifact_hscan.txt`). Thus (a) the divergence and sign change of θ_1 is a numerical artefact, and (b) the point lies beyond the fold, on the partner branch with two relevant directions ($\theta_3 < 0$), not on the branch with three. For $N_D = 22.5$ the old point ($\lambda = -25.15 < \lambda_{\text{fold}} = -22.48$, $s > s_{\text{fold}}$) has the same structure; its exponents were not recomputed. The values of s nevertheless nearly coincide with s_{fold} for $N_D = 22.5$ (1.07219 against 1.07210), because $s(\lambda_*)$ is extremely flat near the minimum; for $N_D = 15$ the difference is larger (1.0344 against 1.0310). The other numbers of the earlier text have similar origins: $s \simeq 1.063$ was the point where, with $N_G = 60$, the branch was lost by a jump (quadrature not converged); 1.073 was where tracking in s alone stops ($d\lambda_*/ds$ grows); 1.10 was a limitation of the tracking. All are superseded by the single description above. The stability matrix is now computed by Richardson extrapolation of central differences with steps 4×10^{-7} and 2×10^{-7} ; it converges to the $h \rightarrow 0$ value shown above.

7.4 $N_D = 27$: run-away without a fold

For $N_D = 27$ the branch has no minimum of s in the range followed. Starting from the fixed point at $s = 1.3$ (where λ_* first rises to a maximum at $\lambda_* \simeq -21.87$ while s continues to decrease), and continuing with decreasing λ_* (scripts `fold_arc.py`, `runaway_trace.py`, $N_G = 240$) one finds, with real exponents and $\arg_{\min} > 0$ throughout,

λ_*	g_*	β_*	s	$\arg_{\min}$	$\theta_1, \theta_2, \theta_3$
-30.1	4.198	0.04851	1.1070	0.0655	505, 3.82, 2.26
-53.6	7.311	0.04917	1.0883	0.0431	1552, 3.84, 1.89
-87.4	11.85	0.04923	1.0849	0.0459	2526, 3.85, 1.55
-109.9	14.88	0.04923	1.0842	0.0506	2948, 3.85, 1.41

so that s decreases monotonically but ever more slowly, while $\lambda_* \rightarrow -\infty$ and $g_* \rightarrow \infty$ and θ_1 grows monotonically (505 $\rightarrow$ 2948) while θ_3 decreases (2.26 $\rightarrow$ 1.41). The run was stopped at $\lambda_* = -117.6$ ($s = 1.0840$). The earlier estimate $s_{\text{run}} \simeq 1.0833$ is compatible with an asymptote near 1.08; *verified* is only that $s > 1.083$ over the range followed. Whether the asymptote is

1.083 is a hypothesis (the data were not extrapolated). For $N_D = 27$ the branch does not end at a fold; it leaves the region in which the truncation is meaningful by run-away.

7.5 Dependence on N_D and comparison with the EH bound

Script `branch_ranges.py` follows the fixed points at fixed s with N_D as an unknown ($N_G = 240$).

- *Branch G* (the Reuter fixed point with matter $N_S = 4$, $N_V = 12$ and N_D switched on from 0): at $s = 1.5$ it is followed up to $N_D = 8.86$, where $\arg_{\min} \rightarrow 0$ and the continuation ends (last point $\lambda_* \simeq 0$, $g_* = 0.712$).
- *Branch S* (Standard-Model type): at $s = 1.5$ ($s = 2$) N_D increases monotonically as λ_* decreases, up to $N_D = 29.92$ (28.76) at $\lambda_* = -606$ (-607), $g_* = 106$ (159), without a sign change of the smallest exponent (i.e. no saddle-node in N_D) before the stopping point; the exponents stay real and positive, e.g. (21.5, 3.78, 2.02) at $s = 1.5$. Towards smaller N_D the exponent structure changes between $N_D = 7.29$ and 6.37 at $s = 1.5$ (between 6.04 and 6.00 at $s = 2$): there $\lambda_* \simeq 0.2$ –0.4, $g_* \simeq 0.3$ –0.4 and the three exponents are $1.85 \pm 2.15i$, 2.27 at $N_D = 7.29$ ($s = 1.5$).

Verified: for $N_D \in (6.4, 8.9)$ at $s = 1.5$ two distinct fixed points coexist, which we call G and S (for the one-generation content $N_D = 7.5$ the G fixed point is $(\lambda_*, g_*, \beta_*) = (0.171, 0.372, 0.00682)$ with $\theta = 2.08 \pm 1.79i$, 10.9; the S point at $N_D = 7.29$ is $(0.220, 0.377, 0.01010)$, $\theta = 1.85 \pm 2.15i$, 2.27), and the two are different zeros of the same flow. We did not find a path connecting G to the Standard-Model fixed point at fixed s while staying in the admissible region (G ends with $\arg_{\min} \rightarrow 0$); this does not prove that none exists.

For comparison, at the EH level with the optimised cutoff ($a = 1$) the fixed point for $N_S = 4$, $N_V = 12$ exists as long as $14 - X > 0$, i.e. $N_D < 29$, and runs away as $N_D \rightarrow 29$ (e.g. $\lambda_* = -4.4, -11.5, -38.1, -3973$ for $N_D = 22.5, 26, 28, 28.99$; Appendix A). The R^2 branch S shows the same pattern (fixed point pushed to $\lambda_* \rightarrow -\infty$, $g_* \rightarrow \infty$ as N_D grows towards ~ 29 –30), with an s -dependent limiting value. *That the limiting N_D coincides with the EH value $X = 14$ is not claimed:* the R^2 values 29.9 and 28.8 are those reached at the stopping point $\lambda_* = -600$, where N_D is still increasing, and the content-dependence of a run-away limit has been checked only for $N_S = 4$, $N_V = 12$.

7.6 Other matter contents

Table 6 lists, for ten contents, the EH-level class from the book’s 50-theory table (nine regulators) and what the R^2 continuation from pure gravity finds (script `r2_contents.py`, $N_G = 60$, adequate for the G-type points, whose pure-gravity limit agrees with $N_G = 240$ to the digits shown). For the contents in the upper rows the verdicts agree: where the EH level has a robust fixed point at small couplings, the R^2 truncation has a fixed point of Reuter type with a complex pair and three relevant directions. For the two-generation and Standard-Model contents the EH fixed point lies outside the conventional range, and the R^2 truncation has a fixed point of a different type (S), also at large couplings, with real exponents. Its two lower exponents (3.64, 1.90 at $s = 1.5$) are of the size of the EH exponents of the same content (3.92, 2.04 for the exponential regulator with $a = 1$; 3.93, 2.08 for the optimised regulator with $a = 1$), and the additional large exponent θ_1 is the one associated with β . This resemblance is an observation, not an identification of the two fixed points.

7.7 What happens at $s = 1$

The member $s = 1$ of the family is the exponential regulator with $a = 1$ of the EH analysis, at which the EH fixed point exists at $(g_*, \lambda_*) = (1.71, -11.2)$ with exponents (2.04, 3.92) (Appendix A). In the R^2 truncation the S-type fixed point does not exist at $s = 1$ on the branch fol-

content	EH class (book)	R^2 , G-type	fixed point at $s = 1/1.5/2$	θ at $s = 1.5$
pure gravity	robust	yes/yes/yes		$2.16 \pm 3.43i$, 23.4
Higgs doublet ($N_S=4$)	robust	yes/yes/yes		$2.91 \pm 3.83i$, 31.8
SM gauge group ($N_V=12$)	robust	yes/yes/yes		$3.85 \pm 2.80i$, 43.4
gauge + Higgs	fragile (exp. $a=0.5$)	yes/yes/yes		$4.23 \pm 2.89i$, 48.0
QED ($N_D=1$, $N_V=1$)	robust	yes/yes/yes		$1.79 \pm 3.10i$, 18.3
SM, one generation ($N_D=7.5$)	robust, range	trusted	yes/yes/yes	$2.08 \pm 1.79i$, 10.9
SM, two generations ($N_D=15$)	robust, range	outside	no G ($N_D > 8.86$ at $s=1.5$); S-type fixed point exists down to $s_{\text{fold}} = 1.031$	–
Standard Model ($N_D=22.5$)	robust, range	outside	no G; S-type for $s > 1.0721$	15.3, 3.64, 1.90
SM, four generations ($N_D=30$)	absent		not found (not a proof of absence)	–
SM fermions only	absent		not found (not a proof of absence)	–

Table 6: EH-level classification against the R^2 continuation. “Not found” means that the continuation failed and that the Newton search from a grid of seeds did not locate a zero; the grid search is weak (Section 7.7). The EH-level “fragile” verdict for gauge+Higgs rests on a single member of the nine regulators (exponential, $a = 0.5$), as noted in the book.

lowed: it ends at $s_{\text{fold}} = 1.0721$. A Newton search from a grid of 350 seeds at $s = 1$ and $s = 1.05$ found no zero. This search is weak: at $s = 1.5$ it finds the known fixed point from one seed in 350, and at $s = 1.1$, where the fixed point exists, it finds nothing (`out/multistart_*.txt`). Hence the non-detection at $s = 1$ has no evidential value on its own; the only firm statement is that the branch ending at s_{fold} does not reach $s = 1$. We cannot say whether another zero exists at $s = 1$.

8 Discussion

8.1 What is established

Verified (reproduced by scripts in the accompanying code).

1. Analytic relations of LR reproduced to $\leq 5 \times 10^{-7}$; pure-gravity fixed point and exponents at $s = 1$ reproduced; the printed (B7)–(B9) are consistent with (A10), (A28) and with LR’s $\theta_3 = 28.8$; the sign of $h_1 \tilde{\Psi}$ in (B2) is a misprint.
2. Pure-gravity fixed point with a complex pair at all sampled $s \in [1, 2.5]$.
3. Matter coefficients: a_2 values on S^4 (29, 11, -4 , -62 over 2160); consistency of X, Y with the EH-level matter terms for the Litim profile.
4. For Standard-Model content a fixed point with three real positive exponents exists for $s > s_{\text{fold}} = 1.07210$ ($N_G \geq 480$) on the branch followed; at $N_D = 15$, $s_{\text{fold}} = 1.03097$. At the fold θ_{min} crosses zero, $\theta_1 \sim 10^3$ – 10^4 , $\arg_{\text{min}} \sim 5 \times 10^{-3}$.
5. The earlier “edges” 1.0344, 1.0722 come from a finite-difference artefact and lie on the partner branch.

Quoted from checked sources. Definition of the truncation, the flow equations and the regulator family (LR); the EH-level classification (book, reproduced in Appendix A for the Standard Model by `eh_sm_by_regulator.py`).

Hypothesis / open.

1. Whether the on-sphere a_2 of matter correctly gives the matter contribution to the R^2 coupling (the projection on S^4 cannot separate R^2 from the other quadratic invariants).
2. Whether the asymptote of $s(\lambda_*)$ for $N_D = 27$ is 1.083.
3. Whether the S-type fixed point is connected to the Reuter fixed point at fixed s , and whether other zeros exist at $s < s_{\text{fold}}$.
4. Gauge dependence ($\alpha = 1$ only), parametrisation dependence, the choice of the family (only the exponential family with $R^{(0)}(0) = 1$ is used), and contributions of $R_{\mu\nu}^2$ or C^2 (not included).
5. The Weyl-gauged QDG sector is not in the flow; whether it modifies it is an open problem.

8.2 What this means for the EH result

The check asked in the introduction gives a mixed answer. For contents at small couplings the R^2 truncation agrees with the EH level. For the Standard-Model content, the R^2 truncation finds a fixed point with real, positive exponents for $s \geq 1.5$ ($s > 1.0721$ in the sampled range) at couplings of the same size as at the EH level, but with an exponent of order 15 (at $s = 1.5$) rising to 10^3 at the lower end, and it has no counterpart at $s = 1$, where the EH level has one. We conclude that the EH-level existence is not contradicted at the regulators where the R^2 fixed point exists, that it is not confirmed at $s = 1$, and that the extreme location of the fixed point makes both results unreliable as statements about the Standard Model. We do *not* claim that the Standard Model coupled to gravity is asymptotically safe, nor that it is not. The quantitative statements of this paper are statements about this truncation, gauge and regulator family.

8.3 Conflicts of interest

The author is the Editor-in-Chief of the Journal of Advanced Gravitational Physics, in which this paper is published. This is a structural conflict of interest; the author has recused himself from the editorial handling of this manuscript.

9 Conclusions

In the R^2 truncation with the exponential family (9), pure gravity has a fixed point at all sampled $s \in [1, 2.5]$. With Standard-Model matter, a fixed point with three real positive exponents exists for $s > 1.0721$ ($N_D = 22.5$; 1.0310 for $N_D = 15$) and ends at a saddle-node, not at a divergence of an exponent; for $N_D = 27$ it runs away to infinite couplings instead, with $s > 1.083$ over the range followed. The earlier values 1.063, 1.073, 1.10, 1.0344, 1.0722 and 1.0833 are replaced by this single description: the threshold “ $s \gtrsim 1.10$ ” of the earlier abstract was a limitation of the tracking, and the “edges” were an artefact. The EH-level robustness of the Standard-Model content is neither contradicted nor confirmed at $s = 1$. Gauge dependence, other regulator families, the C^2 extension and the QDG-specific fields are left open.

A EH-level numbers used for comparison

Script `eh_level/eh_sm_by_regulator.py` (using `rre.py` of the book) gives for $X = 1$, $Y = -62$ the following EH fixed points (g_*, λ_*) with exponents θ : optimised $a = 0.5$: (4.318, -3.619), (2.06, 3.927); $a = 1$: (3.217, -4.393), (2.083, 3.926); $a = 2$: (2.523, -5.318), (2.111, 3.922); exponential $a = 0.5$: (2.527, -9.136), (2.02, 3.915); $a = 1$: (1.711, -11.202), (2.035, 3.920); $a = 2$: (1.202, -13.790), (2.052, 3.923). The classification of the 50

contents is in `eh_level/book_rre_50_theories.csv` (robust 28, fragile 13, absent 9). The scan in N_D at $N_S = 4$, $N_V = 12$ (`eh_sm_ND_scan.py`) gives $(\lambda_*, g_*) = (0.072, 0.648), (-1.043, 1.469), (-4.393, 3.217), (-11.55, 6.96), (-38.1, 20.8), (-77.8, 41.6), (-396, 208), (-397, 208)$ for $N_D = 7.5, 15, 22.5, 26, 28, 28.5, 28.9, 28.99$, and none for $N_D \geq 29.01$.

B Implementation and numerical checks

All numbers come from NumPy/SciPy scripts in the folder `code/` (no symbolic package is used); `code/README.txt` lists script and output file for each table. Gauss–Legendre quadrature with N_G nodes per segment; Newton iteration with a Jacobian step 10^{-7} ; stability matrix by Richardson extrapolation of central differences with steps 4×10^{-7} and 2×10^{-7} (a single step $h = 10^{-5}$ gives spurious results close to a pole of η , Section 7.3). The quadrature converges at $N_G \geq 480$ near the fold ($N_G = 60$ was found insufficient on branch S with the earlier protocol); pure-gravity and G-type results are converged at $N_G = 60$.

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